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On graded fuzzy 1-absorbing primary hyperideals

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Abstract. Let G be a group with identity e and R be a graded commutative multiplicative hyperring. In this article, we aim to introduce the concepts of graded fuzzy radical of a graded fuzzy hyperideal and graded fuzzy primary hyperideals of a graded hyperring. Moreover, we introduce and study the notion of graded fuzzy 1-absorbing primary hyperideals of R as a generalization of graded fuzzy primary hyperideals and we give its relationship with graded fuzzy 2-absorbing primary hyperideals. Many properties and characterizations of graded fuzzy 1-absorbing primary hyperideals of R are stated. Then, we investigate the notions of graded fuzzy strongly 1-absorbing primary hyperideals, graded fuzzy weakly completely primary hyperideals and graded fuzzy weakly completely 1-absorbing primary hyperideals. We obtain some basic properties and new results of these cases of structures and we state the relationship between them.

1. Introduction

Fuzzy sets and hyperstructures were introduced by Zadeh [40] in 1965, and by Marty [30], in 1934, respectively, and are now used in the world both from the theoretical point of view and for their many applications. Rosenfield applied the concept of fuzzy sets in the group theory [37]. In commutative algebra, W. J. Liu [26, 27] explained about improvement approach of the algebraic structure of fuzzy sets by initiating the notion of fuzzy subring, fuzzy normal subgroup along with the confection of fuzzy sets, and introduced the notion of a fuzzy ideal of a ring. Malik and Mordeson presented the direct sum of fuzzy rings and fuzzy ideals [29]. Eslami explained a concept of graded fuzzy rings on natural numbers by direct sum of fuzzy subgroups and then found a way for fuzzification polynomial rings [15]. Since then several authors have obtained interesting results on L-fuzzy ideals of rings and L-fuzzy modules. For a comprehensive survey of the literature on these developments see [28, 38, 41].

Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Hyperstructures have many applications in several sectors of both pure and applied mathematics, for instance in geometry, lattices, cryptography, automata, graphs and hypergraphs, fuzzy set, probability and rough set theory, and so on (see [12, 13]). The notion of hyperrings was introduced by M. Krasner in 1983, where the addition is a hyperoperation, while the

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multiplication is an operation [24]. K. Hila *et al*. in [23] introduced and studied (k, n)-absorbing hyperideals in Krasner (m, n)-hyperrings. Multiplicative hyperrings are an important class of algebraic hyperstructures which are a generalization of rings, initiated the study by Rota in 1982, where the multiplication is a hyperoperation, while the addition is an operation [36]. Procesi and Rota introduced and studied in brief the prime hyperideals of multiplicative hyperrings [32–35] and this idea is further generalized in a paper by Dasgupta [14]. R. Ameri *et al*. in [4] described multiplicative hyperrings of fractions and coprime hyperideals. The notion of n-absorbing ideals over commutative rings which is a generalization of prime ideals has been introduced and investigated by A. F. Anderson and A. Badawi in [7]. In [18], P. Ghiasvand introduced and studied the concept of 2-absorbing hyperideals. Then M. Anbarloei in [5] studied the concepts of 2-absorbing and 2-absorbing primary hyperideals of a multiplicative hyperring. 1-absorbing prime hyperideals in a multiplicative hyperring has been studied by M. Anbarloei [6]. Furthermore, in [20], has been studied 1-absorbing prime avoidance theorem in multiplicative hyperrings. Also, many researchers have observed generalizations of prime hyperideals in multiplicative hyperrings. The principal notions of algebraic hyperstructure theory can be found in [12, 13].

The study of graded rings arises naturally out of the study of affine schemes and allows them to formalize and unify arguments by induction [39]. However, this is not just an algebraic trick. The concept of grading in algebra, in particular graded modules is essential in the study of the homological aspect of rings. Much of the modern development of commutative algebra emphasizes graded rings. Gradings appear in many circumstances, both at elementary and advanced levels. Studying graded prime ideals and their generalizations is important for several reasons. First, graded prime ideals are a natural generalization of prime ideals in commutative algebra, algebraic number theory and algebraic geometry, which are fundamental concepts in mathematics. Additionally, graded prime ideals are related to other important mathematical concepts such as graded rings and modules, which are important in areas such as algebraic geometry and algebraic topology. Understanding these concepts can help to provide a deeper understanding of the underlying structures in these areas of mathematics. Finally, the study of graded prime ideals and their generalizations can also have practical applications in areas such as computer science and engineering. For example, the theory of graded prime ideals can be used to study error-correcting codes and cryptography. Overall, studying graded prime ideals and their generalizations is important for both theoretical and practical reasons, and can help to deepen our understanding of many areas of mathematics and its application to other fields. In recent years, rings with a group-graded structure have become increasingly important and consequently, the graded analogues of different concepts are widely studied (see [2, 3, 11, 21, 31]). Also, the theory of graded hyperrings can be considered as an extension theory of hyperrings [16, 17, 21, 22]. Moreover, Fuzzy prime hyperideals play an important role in fuzzy multiplicative hyperring theory. The notion of 1-absorbing prime ideals which is another extension of prime ideals was introduced in [42] and 1-absorbing primary and weakly 1-absorbing primary ideals were investigated in [8, 9]. R. Abu-Dawwas *et al*. in [1] studied graded 1-absorbing prime ideals and in [10], introduced the concept of graded weakly 1-absorbing primary ideals which is a generalization of graded 1-absorbing primary ideals. In [19], we introduced and studied the notions of graded fuzzy multiplicative hyperrings and graded fuzzy n-absorbing hyperideals of R. In this paper, we study the concepts of graded fuzzy radical of a graded fuzzy hyperideal of a multiplicative hyperring as a generalization of fuzzy radical of a fuzzy ideal of a ring and graded fuzzy primary hyperideals of a graded hyperring. We introduce and study the notion of graded fuzzy 1-absorbing primary hyperideals of R as a generalization of fuzzy primary hyperideals. Many properties, examples, and characterizations of graded fuzzy 1-absorbing primary hyperideals of R are introduced. Moreover, we investigate the concepts of graded fuzzy strongly 1-absorbing primary hyperideals, and graded fuzzy weakly completely 1-absorbing hyperideals. We obtain some basic properties and new results of this case of structures and we state the relationship between them.

2. Preliminaries

In this section, we would like to reproduce some definitions and examples which were proposed earlier. The second type of hyperring was introduced by R. Rota [36] in 1982. The multiplication is a hyperoperation, while the addition is an operation, that is why she called it a multiplicative hyperring. Let us give the definition.

Let R be a non-empty set. By $\wp^*(R)$, we mean the set of all non-empty subsets of R. Let $\circ$ be a hyperoperation from $R \times R$ to $\wp^*(R)$. $(R, +, \circ)$ is a multiplicative hyperring, if it has the following properties:

(i) $(R, +)$ is an abelian group;

(ii) $(R, \circ)$ is a hypersemigroup;

(iii) For all $a, b, c \in R$, $a \circ (b + c) \subseteq a \circ b + a \circ c$ and $(b + c) \circ a \subseteq b \circ a + c \circ a$;

(iv) $a \circ (-b) = (-a) \circ b = -(a \circ b)$.

If in (*iii*) we have equalities instead of inclusions, then we say that the multiplicative hyperring is strongly distributive.

Here, we mean a hypersemigroup by a non-empty set R with an associative hyperoperation $\circ$, i.e.,

$$a \circ (b \circ c) = \bigcup_{t \in (b \circ c)} a \circ t = \bigcup_{s \in (a \circ b)} s \circ c = (a \circ b) \circ c$$

for all $a, b, c \in R$.

Further, if R is a multiplicative hyperring with $a \circ b = b \circ a$ for all $a, b \in R$, then R is called a commutative multiplicative hyperring. Let us give an example.

Example 2.1. *Let K be a field and V be a vectorial space over K. If for all $a, b \in K$ we denote by (a, b) the subspace generated by the subset $\{a, b\}$ of V, then we can consider the following hyperoperation on V: for all $a, b \in V, a \circ b = (a, b)$. It follows that $(V, +, \circ)$ is a multiplicative hyperring, which is not strongly distributive.*

In what follows, we give some definitions and properties of a multiplicative hyperring that we need in this article.

Definition 2.2. *[13] (a) Let $(R, +, \circ)$ be a multiplicative hyperring and S be a non-empty subset of R. Then S is said to be a subhyperring of R if $(S, +, \circ)$ is itself a multiplicative hyperring.*

(b) A subhyperring I of a multiplicative hyperring R is a hyperideal of $(R, +, \circ)$ if $I - I \subseteq I$ and for all $x \in I$, $r \in R$; $x \circ r \cup r \circ x \subseteq I$.

Definition 2.3. *[14] (a) A proper hyperideal P of a multiplicative hyperring R is said to be a prime hyperideal of R, if for any $a, b \in R$, $a \circ b \subseteq P$, then $a \in P$ or $b \in P$.*

(b) A proper hyperideal Q of a multiplicative hyperring R is said to be a primary hyperideal of R, if for any $a, b \in R$, $a \circ b \subseteq Q$, then $a \in Q$ or $b^n \subseteq Q$ for some $n \in \mathbb{N}$.

Example 2.4. *Let $R = (\mathbb{Z}[i], +, \cdot)$ be the Gaussian integers ring. Consider the multiplicative hyperring $(R_A, +, \circ) = (\mathbb{Z}[i], +, \circ) = \{a + bi \mid a, b \in \mathbb{Z}\}$ with $A = \{-1, 3\}$, where $R_A = R$ and for all $x, y \in R_A$, $x \circ y = \{x \cdot a \cdot y : a \in A\}$. Then, $(R_A, +, \circ)$ is a multiplicative hyperring. The hyperideal $P = 2R = \{-2a - 2bi, 6a + 6bi : a, b \in \mathbb{Z}\}$ is a prime (primary) hyperideal of R.*

This paragraph reviews the concepts and notations of fuzzy sets in commutative multiplicative hyperrings which we need in the following.

Definition 2.5. *[28] (a) A fuzzy subset μ in a set X is a function $\mu : X \longrightarrow [0, 1]$. We denote by $F(X)$ the set of all fuzzy subsets of X.*

(b) Let μ and ν be fuzzy subsets of X. We say that μ is a subset of ν, and $\mu \subseteq \nu$, if and only if $\mu(x) \leq \nu(x)$, for all $x \in X$.

(c) Let $\mu \in F(X)$ and $t \in [0, 1]$. Then the set $\mu^t = \{x \in X \mid \mu(x) \geq t\}$ is called the t-level subset of X with respect to μ.

Definition 2.6. *[28] (a) Let $x \in X$ and $r \in (0,1]$. A fuzzy point, written as x^r, is defined to be a fuzzy subset of X, given by*

$$x^r(y) = \begin{cases} r, & y = x; \\ 0, & otherwise. \end{cases}$$

If x^r is a fuzzy point of X and $x^r \subseteq \mu$, where $\mu \in F(X)$, then we write $x^r \in \mu$.
(b) For $A \subseteq X$ the characteristic function of A, $\chi_A \in F(X)$, is defined by

$$\chi_A(x) = \begin{cases} 1, & x \in A; \\ 0, & otherwise. \end{cases}$$

Definition 2.7. *[41] Let μ and ν be two fuzzy subsets of a commutative multiplicative hyperring R. Define the product $\mu\nu$ and the composition $\mu \circ \nu$ respectively as follows: For all $w \in R$,*

$$(\mu\nu)(w) = \sup\{\inf_{i=1}^{n}\{\mu(r_i) \wedge \nu(s_i)\} \mid r_i, s_i \in R, n \in \mathbb{N}, w \in \textstyle\sum_{i=1}^{n} r_i s_i\},$$

$$(\mu \circ \nu)(w) = \sup\{\mu(r) \wedge \nu(s) \mid r, s \in R, w \in rs\}.$$

Definition 2.8. *[41] Let μ be a fuzzy subset in a commutative multiplicative hyperring R with identity 1 that $\mu(0) = 1$.*
(a) μ is said to be a fuzzy subgroup of R if for all $x, y \in R$, we have

$$\mu(x - y) \geq min\{\mu(x), \mu(y)\}.$$

(b) We call fuzzy subgroup μ a fuzzy hyperring of R if for all $x, y \in R$, we have

$$\inf_{z \in x \circ y} \mu(z) \geq \min\{\mu(x), \mu(y)\}.$$

(c) We call fuzzy subgroup μ a fuzzy hyperideal of R if for all $x, y \in R$, we have

$$\inf_{z \in x \circ y} \mu(z) \geq \max\{\mu(x), \mu(y)\}.$$

Definition 2.9. *[29] Let $\{\mu_i \mid i \in I\}$ be a family of fuzzy subsets on a commutative multiplicative hyperring R. Fuzzy subset $\sum_{i\in I} \mu_i$ on R is defined as below,*

$$\left(\textstyle\sum_{i\in I} \mu_i\right)(x) = \sup\{\inf\{\ \mu_i(x_i) \mid x = \textstyle\sum_{i\in I} x_i,\ \forall i \in I\}\}\ \ \forall x \in R.$$

Remark 2.10. *Let $\{\mu_i \mid i \in I\}$ be a family of fuzzy subgroups, fuzzy hyperrings or fuzzy hyperideals on a commutative multiplicative hyperring R, then easily it can be shown that $\sum_{i\in I} \mu_i$ is a fuzzy subgroup, a fuzzy hyperring or a fuzzy hyperideal on R, respectively. For every $x \in R$, if $i = j$, we set $x_j = x$ and if $i \neq j$, $x_j = 0$, then by the definition of $\sum_{i\in I} \mu_i$, we have*

$$\mu_i \subseteq \textstyle\sum_{j\in I} \mu_j, \quad \forall j \in I.$$

Definition 2.11. *[29] Let μ_i and μ with $i \in I$ be fuzzy subsets on a commutative multiplicative hyperring R. Then R is called the weak direct sum of $\{\mu_i \mid i \in I\}$, if $\mu = \sum_{i\in I} \mu_i$ and $\mu_j \cap \sum_{i\neq j} \mu_i = 1_0$ such that,*

$$1_0(x) = \begin{cases} 1, & x = 0; \\ 0, & x \neq 0. \end{cases}$$

In this case, $\mu = \oplus_{i\in I}\mu_i$ is written.

In this paragraph, we present the notion of graded multiplicative hyperrings and present an example.

Definition 2.12. *[21] Let G be a group with identity element e. A multiplicative hyperring R is called a G-graded multiplicative hyperring if there exists a family $\{R_g\}_{g\in G}$ of additive subgroups of R indexed by the elements $g \in G$ such that $R = \bigoplus_{g\in G} R_g$ and $R_gR_h \subseteq R_{gh}$ for all $g, h \in G$ where $R_gR_h = \bigcup\{r_g \circ r_h : r_g \in R_g, r_h \in R_h\}$.*

An element of a graded hyperring R is called homogeneous if it belongs to $\bigcup_{g\in G} R_g$ and this set of homogeneous elements is denoted by $H(R)$. If $x \in R_g$ for some $g \in G$, then we say that x is of degree g, and it is denoted by x_g. If $x \in R$, then there exist unique elements $x_g \in H(R)$ such that $x = \sum_{g\in G} x_g$.

Example 2.13. *Let $G = (\mathbb{Z}_2, +)$ be the cyclic group of order 2 and $R = \{a, b, c, d\}$. Consider the multiplicative hyperring $(R, +, \circ)$, where operation + and hyperoperation ∘ defined on R as follow:*

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

$\circ$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a,d\}$	$\{a,c\}$	$\{a,b\}$
c	$\{a\}$	$\{a,c\}$	$\{a\}$	$\{a,c\}$
d	$\{a\}$	$\{a,b\}$	$\{a,c\}$	$\{a,d\}$

It is easy to see that $R_0 = \{a, d\}$ and $R_1 = \{a, b\}$ are subgroups of $(R, +)$. We have $a = a + a$, $b = a + b$, $c = d + b$ and $d = d + a$ and these forms are unique and also we have $R_0R_0 \subseteq R_0$, $R_0R_1 \subseteq R_1$, $R_1R_0 \subseteq R_1$ and $R_1R_1 \subseteq R_0$. Hence, $R = R_0 \bigoplus R_1$. Therefore (R, G) is a graded hyperring and $H(R) = \{a, b, d\}$.

Throughout this paper, R is a commutative graded multiplicative hyperring with non-zero identity 1.

3. Graded fuzzy 1-absorbing primary hyperideals

In this section, the basic properties of graded fuzzy 1-absorbing primary hyperideals are studied. Firstly, we will give the structure of graded fuzzy hyperrings and graded fuzzy primary hyperideals of a graded hyperring.

Definition 3.1. *[19] Let G be a group with identity element e and μ be a fuzzy hyperring on graded multiplicative hyperring $(R, +, \circ)$. A fuzzy hyperring μ is called a G-graded fuzzy hyperring if there exists a family $\{\mu_g\}_{g\in G}$ of fuzzy subgroups of R such that $\mu = \bigoplus_{g\in G} \mu_g$ and $\mu_g\mu_h \subseteq \mu_{gh}$ for all $g, h \in G$, where*

$$(\mu_g\mu_h)(x) = \sup\{\inf_{i=1}^{n}\{\mu_g(r_i) \wedge \mu_h(s_i)\} \mid r_i, s_i \in R, n \in \mathbb{N}, x \in \textstyle\sum_{i=1}^{n} r_i \circ s_i\}.$$

For any $x \in R$, $\mu_g(x) = \mu(x_g)$ where $x = \sum_{g\in G} x_g$.

Example 3.2. *[19] Let $(R, +, \cdot)$ be a ring. Then corresponding to every subset $A \in P^*(R)(|A| \geq 2)$, there exists a multiplicative hyperring with absorbing zero $(R_A, +, \circ)$, where $R_A = R$ and for any $\alpha, \beta \in R_A$, $\alpha\circ\beta = \{\alpha\cdot a\cdot\beta : a \in A\}$. Let $R = (\mathbb{Z}[i], +, \cdot)$ be the Gaussian integers ring and $G = (\mathbb{Z}_2, +)$ be the cyclic group of order 2. Consider the multiplicative hyperring $(R_A, +, \circ) = (\mathbb{Z}_4[i], +, \circ) = \{a + bi \mid a, b \in \mathbb{Z}_4\}$ with $A = \{1, 2\}$, where $R_A = R$ and for any $x, y \in R_A$, $x \circ y = \{x \cdot a \cdot y : a \in A\}$. Then, $(R_A, +, \circ)$ is a G-graded multiplicative hyperring with $R_0 = \mathbb{Z}_4$ and $R_1 = i\mathbb{Z}_4$ and $R_A = R_0 \bigoplus R_1$. Consider the fuzzy multiplicative hyperring μ by definition*

$$\mu(x) = \begin{cases} 1 & x = 0; \\ \frac{3}{4} & x \in \{1, 2, 3\}; \\ \frac{1}{4} & x \in \mathbb{Z}_4[i] - \{0, 1, 2, 3\}. \end{cases}$$

However, an easy computation shows that μ is a graded fuzzy multiplicative hyperring of R such that $\mu = \mu_0 \bigoplus \mu_1$ where

$$\mu_0(x) = \begin{cases} 1 & x \in i\mathbb{Z}_4; \\ \frac{3}{4} & x \in \mathbb{Z}_4[i] - i\mathbb{Z}_4 \end{cases}$$

and

$$\mu_1(x) = \begin{cases} 1 & x \in \mathbb{Z}_4; \\ \frac{1}{4} & x \in \mathbb{Z}_4[i] - \mathbb{Z}_4. \end{cases}$$

Example 3.3. *Let $G = (\mathbb{Z}_2, +)$ be the cyclic group of order 2 and $R = \{a, b, c, d\}$. Consider the graded multiplicative hyperring $(R, +, \circ)$, where operation $+$ and hyperoperation $\circ$ are defined on R as follows:*

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

$\circ$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a,d\}$	$\{a,c\}$	$\{a,b\}$
c	$\{a\}$	$\{a,c\}$	$\{a\}$	$\{a,c\}$
d	$\{a\}$	$\{a,b\}$	$\{a,c\}$	$\{a,d\}$

Consider the fuzzy hyperring μ by definition

$$\mu(a) = 1,\ \mu(b) = \mu(c) = \tfrac{1}{4},\ \mu(d) = \tfrac{3}{4}.$$

Then $\mu = \mu_0 \bigoplus \mu_1$ is a G-graded fuzzy multiplicative hyperring such that

$$\mu_0(a) = \mu_0(b) = 1,\ \ \mu_0(c) = \mu_0(d) = \tfrac{3}{4},$$

$$\mu_1(a) = \mu_1(d) = 1,\ \ \mu_1(b) = \mu_1(c) = \tfrac{1}{4}$$

and

$$\mu_0\mu_0 \subseteq \mu_0,\ \ \mu_0\mu_1 \subseteq \mu_1,\ \mu_1\mu_0 \subseteq \mu_1,\ \ \mu_1\mu_1 \subseteq \mu_0.$$

Definition 3.4. *[19] (a) A fuzzy set η of the graded hyperring R is said to be a graded fuzzy set of R if $\eta_g(r) \geq \eta(r)$ for all $g \in G$ and $r \in R$.*

(b) Let η be a fuzzy hyperideal of R. Then η is said to be a graded fuzzy hyperideal of R if η is graded as a fuzzy set of R.

It is easy to see that a fuzzy hyperideal η of R is graded if and only if $\eta = \eta_G$, where $\eta_G = \eta \cap (\cap_{g\in G}\eta_g)$. More generally, η_G is the largest graded fuzzy hyperideal of R contained in η.

Example 3.5. *Let $(R_A, +, \circ) = (\mathbb{Z}_4[i], +, \circ) = \{a + bi \mid a, b \in \mathbb{Z}_4\}$ be the graded multiplicative hyperring with $A = \{1, 2\}$. Consider the fuzzy hyperideal η by definition*

$$\eta(x) = \begin{cases} 1 & x = 0; \\ \frac{2}{3} & x \in \langle 2\rangle - \{0\}; \\ \frac{1}{5} & x \in \mathbb{Z}_4[i] - \langle 2\rangle. \end{cases}$$

Then η is a graded fuzzy hyperideal of R. For this purpose, it is enough to show η is graded as a fuzzy set of R. We have

$$\eta_0(0) = 1 \geq \eta(0) = 1,$$

$$\eta_0(x) = \tfrac{1}{5} \geq \eta(x) = \tfrac{1}{5}\ \ \forall x \in \{1, 3, 1 + i, 3 + i, 1 + 2i, 3 + 2i, 3 + 3i, 1 + 2i\},$$

$$\eta_0(y) = \tfrac{2}{3} \geq \eta(y) = \tfrac{2}{3}\ \ \forall y \in \{2, 2 + 2i\},$$

$$\eta_0(z) = 1 \geq \eta(z) = \tfrac{1}{5}\ \ \forall z \in \{i, 2i, 3i\},$$

$$\eta_0(t) = \tfrac{2}{3} \geq \eta(t) = \tfrac{1}{5}\ \ \forall t \in \{2 + i, 2 + 3i\},$$

and

$$\eta_1(0) = 1 \geq \eta(0) = 1, \eta_1(2) = 1 \geq \eta(2) = \tfrac{2}{3},$$

$$\eta_1(2+2i) = \tfrac{2}{3} \geq \eta(2+2i) = \tfrac{2}{3},$$

$$\eta_1(x) = \tfrac{1}{5} \geq \eta(x) = \tfrac{1}{5} \quad \forall x \in \{i, 3i, 1+i, 2+i, 3+i, 1+3i, 2+3i, 3+3i\},$$

$$\eta_1(z) = 1 \geq \eta(z) = \tfrac{1}{5} \quad \forall z \in \{1, 3\},$$

$$\eta_1(t) = \tfrac{2}{3} \geq \eta(t) = \tfrac{1}{5} \quad \forall t \in \{1+2i, 3+2i\}.$$

Definition 3.6. (a) *Let I be a graded hyperideal of a graded multiplicative hyperring R. The graded radical I is denoted by $Grad(I)$ and defined by*

$$Grad(I) = \{x = \sum_{g\in G} x_g \in R \mid \forall g \in G, x_g^{n_g} = \underbrace{x_g \circ \cdots \circ x_g}_{n_g \ times} \subseteq I \text{ for some } n_g \in \mathbb{N}\}.$$

(b) *Let λ be a graded fuzzy hyperideal of R. The graded fuzzy radical λ is denoted by $FGrad(\lambda)$ and defined by*

$$\begin{aligned}(FGrad(\lambda))(x) &= \sup\{\inf\{\lambda({x_g}^{n_g}) \mid x = \sum_{g\in G} x_g, \forall g \in G, n_g \in \mathbb{N}\}\} \\ &= \sup_{n_g\geq 1}\{\inf_{t\in \underbrace{x_g \circ \cdots \circ x_g}_{n_g \ times}} \lambda(t) \mid x = \sum_{g\in G} x_g, \forall g \in G, n_g \in \mathbb{N}\} \\ &= \bigvee_{n_g\geq 1} \bigwedge_{t\in \underbrace{x_g \circ \cdots \circ x_g}_{n_g \ times}} \lambda(t).\end{aligned}$$

Example 3.7. *Consider $\mathbb{Z}_2$-graded multiplicative hyperring $\mathbb{Z}_A[i]$ with $A = \{1, 2, 4\}$. Take the graded fuzzy hyperideal λ by definition*

$$\lambda(x) = \begin{cases} 1 & x = 0; \\ \frac{3}{4} & x \in \langle 8\rangle - \{0\}; \\ \frac{1}{3} & x \in \mathbb{Z}_A[i] - \langle 8\rangle. \end{cases}$$

It is easy to see that

$$FGrad(\lambda)(x) = \begin{cases} 1 & x = 0; \\ \frac{3}{4} & x \in \langle 2\rangle - \{0\}; \\ \frac{1}{3} & x \in \mathbb{Z}_A[i] - \langle 2\rangle. \end{cases}$$

Definition 3.8. *Let λ be a non-constant graded fuzzy hyperideal of R. Then λ is said to be a graded fuzzy primary hyperideal if for any fuzzy points $x_g^r, y_h^s \in FH(R)$,*

$$x_g^r \circ y_h^s \subseteq \lambda \text{ implies that either } x_g^r \in \lambda \text{ or } y_h^s \in FGrad(\lambda).$$

Example 3.9. *Consider $\mathbb{Z}_2$-graded multiplicative hyperring $R = \mathbb{Z}_A[i]$ with $A = \{1, 4\}$ and the graded fuzzy hyperideal ζ by definition*

$$\zeta(x) = \begin{cases} 1 & x = 0; \\ \frac{4}{5} & x \in \langle 2\rangle - \{0\}; \\ \frac{3}{4} & x \in \mathbb{Z}_A[i] - \langle 2\rangle. \end{cases}$$

Then ζ is a graded fuzzy primary hyperideal of R.

Proposition 3.10. *Let λ be a graded fuzzy hyperideal of R. If λ is a graded fuzzy primary hyperideal of R, then for all $x_g, y_h \in H(R)$*

$$\lambda(x_g \circ y_h) > \lambda(x_g) \text{ implies that } \lambda(x_g \circ y_h) \leq FGrad(\lambda)(y_h).$$

Proof. Let $\lambda(x_g \circ y_h) = r > \lambda(x_g)$, so we have $\lambda(x_g \circ y_h) = \bigwedge_{t \in x_g \circ y_h} \lambda(t) = r > \lambda(x_g)$. Then $(x_g \circ y_h)^r = x_g^r \circ y_h^r \subseteq \lambda$ and $x_g^r \notin \lambda$. Since λ is a graded fuzzy primary hyperideal of R, then $y_h^r \in FGrad(\lambda)$. Thus $\lambda(x_g \circ y_h) = \bigwedge_{t \in x_g \circ y_h} \lambda(t) = r \leq FGrad(\lambda)(y_h)$. □

Theorem 3.11. *Let I be a graded hyperideal of R. The characteristic function χ_I is a fuzzy primary hyperideal of R if and only if I is a graded primary hyperideal of R.*

Proof. Let I be a graded primary hyperideal of R. Suppose for $x_g, y_h \in H(R)$, $\chi_I(x_g \circ y_h) > \chi_I(x_g)$. Since $\chi_I(x_g)$ is either zero or 1, we find from $x_g, y_h \in H(R)$, $\chi_I(x_g \circ y_h) > \chi_I(x_g)$ that $\chi_I(x_g) = 0$ and $\chi_I(x_g \circ y_h) = 1$. Hence $x_g \circ y_h \subseteq I$ and $x_g \notin I$. Since I is a graded primary hyperideal,

$$y_h^n = y_h \circ \cdots \circ y_h = \bigcup_{t \in \underbrace{y_h \circ \cdots \circ y_h}_{n \text{ times}}} t \subseteq I$$

for some positive integer n. Hence

$$\chi_I(y_h^n) = \bigwedge_{t \in y_h \circ \cdots \circ y_h} \chi_I(t) = 1 = \chi_I(x_g \circ y_h) = \bigwedge_{t' \in x_g \circ y_h} \chi_I(t'),$$

so that χ_I is a graded fuzzy primary hyperideal of R.

Conversely, assume that χ_I is a graded fuzzy primary hyperideal. Let $x_g \circ y_h \subseteq I$ and $x_g \notin I$. Then $\chi_I(x_g \circ y_h) = \bigwedge_{t' \in x_g \circ y_h} \chi_I(t') = 1$ and $\chi_I(x_g) = 0$. Hence $\chi_I(x_g \circ y_h) > \chi_I(x_g)$. Since χ_I is a graded fuzzy primary hyperideal, there exists a positive integer n such that $\chi_I(y_h^n) \geq \chi_I(x_g \circ y_h)$. Hence $\chi_I(y_h^n) = \bigwedge_{t \in y_h \circ \cdots \circ y_h} \chi_I(t) = 1$. This shows that $y_h^n = y_h \circ \cdots \circ y_h \subseteq I$. Consequently I is a graded primary hyperideal. □

Theorem 3.12. *Let λ be a graded fuzzy hyperideal of R. Then λ is a graded fuzzy primary hyperideal if and only if $\lambda^\alpha = \{x \in H(R) \mid \lambda(x) \geq \alpha\}$ is a graded primary hyperideal of R for all $\alpha \in [0, \lambda(0)]$.*

Proof. For any graded fuzzy hyperideal λ, it is known that λ^α is a graded hyperideal of R. Now suppose λ is a graded fuzzy primary hyperideal of R. Let $x_g \circ y_h \subseteq \lambda^\alpha$ and $x_g \notin \lambda^\alpha$ for $x_g, y_h \in H(R)$. Then $\lambda(x_g \circ y_h) = \bigwedge_{t' \in x_g \circ y_h} \lambda(t') \geq \alpha$ and $\lambda(x_g) < \alpha$. Since λ is a graded fuzzy primary hyperideal, there exists a positive integer n such that $\lambda(y_h^n) = \bigwedge_{t \in y_h \circ \cdots \circ y_h} \lambda(t) \geq \lambda(x_g \circ y_h) = \bigwedge_{t' \in x_g \circ y_h} \lambda(t') \geq \alpha$. Then $y_h^n = \bigwedge_{t \in y_h \circ \cdots \circ y_h} t \subseteq \lambda^\alpha$, so that λ^α is a graded primary hyperideal of R.

Conversely, assume that λ^α is a graded primary hyperideal of R for all $\alpha \in [0, \lambda(0)]$. Suppose $\lambda(x_g \circ y_h) = \bigwedge_{t' \in x_g \circ y_h} \lambda(t') \geq \lambda(x_g)$ where $x_g, y_h \in H(R)$. Let $\lambda(x_g \circ y_h) = \bigwedge_{t' \in x_g \circ y_h} \lambda(t') = \alpha$. Then $\alpha \in [0, \lambda(0)]$, $x_g \circ y_h \subseteq \lambda^\alpha$ and $\alpha > \lambda(x_g)$ implies that $x_g \notin \lambda^\alpha$. From the assumption, there exists a positive integer n such that $y_h^n = \underbrace{y_h \circ \cdots \circ y_h}_{n\ times} \subseteq \lambda^\alpha$. Hence $\lambda(y_h^n) = \bigwedge_{t \in y_h \circ \cdots \circ y_h} \lambda(t) \geq \alpha = \lambda(x_g \circ y_h) = \bigwedge_{t' \in x_g \circ y_h} \lambda(t')$. This proves that λ is a graded primary fuzzy hyperideal of R. □

Definition 3.13. (a) *Let λ be a graded fuzzy hyperideal of R. λ is said to be a graded fuzzy* 1*-absorbing prime hyperideal of R if it is non-constant and if for any non-unit elements $x_g^r, y_h^s, z_k^t \in FH(R)$,*

$$x_g^r \circ y_h^s \circ z_k^t \subseteq \lambda \text{ implies that either } x_g^r \circ y_h^s \subseteq \lambda \text{ or } z_k^t \in \lambda.$$

(*b*) *Let λ be a graded fuzzy hyperideal of R. λ is said to be a graded fuzzy* 1*-absorbing primary hyperideal of R if it is non-constant and if for any non-unit elements $x_g^r, y_h^s, z_k^t \in FH(R)$,*

$$x_g^r \circ y_h^s \circ z_k^t \subseteq \lambda \text{ implies that either } x_g^r \circ y_h^s \subseteq \lambda \text{ or } z_k^t \in FGrad(\lambda).$$

(*c*) *Let λ be a graded fuzzy hyperideal of R. λ is said to be a graded fuzzy* 2*-absorbing primary hyperideal of R if it is non-constant and if for any $x_g^r, y_h^s, z_k^t \in FH(R)$,*

$$x_g^r \circ y_h^s \circ z_k^t \subseteq \lambda \text{ implies that either } x_g^r \circ y_h^s \subseteq \lambda \text{ or } x_g^r \circ z_k^t \subseteq FGrad(\lambda) \text{ or } y_h^s \circ z_k^t \subseteq FGrad(\lambda).$$

It is clear that every graded fuzzy 1-absorbing prime hyperideal of R is a graded fuzzy 1-absorbing primary hyperideal of R.

Example 3.14. *In the $\mathbb{Z}_2$-graded multiplicative hyperring $R = (\mathbb{Z}_A[i], +, \circ)$ with $A = \{1, 2\}$ consider the fuzzy hyperideal*

$$\eta(x) = \begin{cases} 1 & x = 0; \\ \frac{2}{3} & x \in \langle 3 \rangle - \{0\}; \\ \frac{1}{3} & x \in \mathbb{Z}[i] - \langle 3 \rangle \,. \end{cases}$$

However, an easy computation shows that η is a graded fuzzy 1*-absorbing primary hyperideal of R.*

Proposition 3.15. *Every graded fuzzy primary hyperideal of R is a graded fuzzy* 1*-absorbing primary hyperideal of R and every graded fuzzy* 1*-absorbing primary hyperideal is a graded fuzzy* 2*-absorbing primary hyperideal.*

Proof. The proof is straightforward. □

The following example shows that a graded fuzzy 2-absorbing primary hyperideal need not be a graded fuzzy 1-absorbing primary hyperideal of R.

Example 3.16. *In the $\mathbb{Z}_2$-graded multiplicative hyperring $R = (\mathbb{Z}_A[i], +, \circ)$ with $A = \{1, 2, 4\}$ consider the fuzzy hyperideal*

$$\eta(x) = \begin{cases} 1 & x = 0; \\ \frac{3}{5} & x \in \langle 12 \rangle - \{0\}; \\ \frac{1}{3} & x \in \mathbb{Z}[i] - \langle 12 \rangle \,. \end{cases}$$

Then η is a graded fuzzy 2*-absorbing primary hyperideal, but it is not a graded fuzzy* 1*-absorbing primary hyperideal. Because $(2^{\frac{2}{3}} \circ 2^{\frac{2}{3}} \circ 3^{\frac{1}{2}}) \subseteq \eta$, but $2^{\frac{2}{3}} \circ 2^{\frac{2}{3}} \nsubseteq \eta$ and $3^{\frac{1}{2}} \notin FGrad(\eta)$.*

Theorem 3.17. *Let λ be a graded fuzzy* 1*-absorbing primary hyperideal of R. Then $FGrad(\lambda)$ is a graded fuzzy prime hyperideal of R.*

Proof. Let $x_g^r \circ y_h^s \subseteq \lambda$ for some $x_g^r, y_h^s \in FH(R)$. We may assume that x_g^r, y_h^s are non-unit elements of R. Let $n \geq 2$ be an even positive integer such that $(x_g^r \circ y_h^s)^n \subseteq \lambda$. Then $n = 2m$ for some positive integer $m \geq 1$. Since

$$(x_g^r \circ y_h^s)^n = \underbrace{(x_g^r \circ y_h^s) \circ (x_g^r \circ y_h^s) \circ \cdots \circ (x_g^r \circ y_h^s)}_{n \ times} = (x_g^r)^n \circ (y_h^s)^n = (x_g^r)^m \circ (x_g^r)^m \circ (y_h^s)^n \subseteq \lambda$$

and λ is a graded fuzzy 1-absorbing primary hyperideal of R, we conclude that $(x_g^r)^m \circ (x_g^r)^m = (x_g^r)^n \subseteq \lambda$ or $(y_h^s)^n \subseteq \lambda$. Hence $x_g^r \in FGrad(\lambda)$ or $y_h^s \in FGrad(\lambda)$. Thus $FGrad(\lambda)$ is a graded fuzzy prime hyperideal of R. □

Theorem 3.18. *Let λ be a graded fuzzy hyperideal of R. If λ is a graded fuzzy* 1*-absorbing primary hyperideal of R, then λ^α is a graded* 1*-absorbing primary hyperideal of R, for every $\alpha \in [0, \lambda(0)]$ with $\lambda^\alpha \neq R$.*

Proof. Let λ be a graded fuzzy 1-absorbing primary hyperideal of R. Assume that $\alpha \in [0, \lambda(0)]$ with $\lambda^\alpha \neq R$ and $x_g \circ y_h \circ z_k \subseteq \lambda^\alpha$, where $y_h, z_k \in H(R)$. Then $\lambda(x_g \circ y_h \circ z_k) \geq \alpha$. So, we have $x_g^\alpha \circ y_h^\alpha \circ z_k^\alpha = (x_g \circ y_h \circ z_k)^\alpha \subseteq \lambda$. Since λ is a graded fuzzy 1-absorbing primary hyperideal of R, we get

$$(x_g \circ y_h)^\alpha = x_g^\alpha \circ y_h^\alpha \subseteq \lambda \text{ or } z_k^\alpha \in FGrad(\lambda).$$

Thus $x_g \circ y_h \subseteq \lambda^\alpha$ or $z_k \in FGrad(\lambda^\alpha)$. Therefore λ^α is a G-graded 1-absorbing primary hyperideal of R. □

Corollary 3.19. *If λ is a graded fuzzy 1-absorbing primary hyperideal of R, then*

$$\lambda_* = \{x \in H(R) \mid \lambda(x) = \lambda(0)\}$$

is a G-graded 1-absorbing primary hyperideal of R.

Proof. Since λ is a non-constant fuzzy hyperideal of R, then $\lambda_* \neq R$. Now the result follows from the above theorem. □

Theorem 3.20. *Let I be a graded 1-absorbing primary hyperideal of R. Then the fuzzy subset of R defined by*

$$\lambda(x) = \begin{cases} 1, & x \in I \\ 0, & otherwise \end{cases}$$

is a graded fuzzy 1-absorbing primary hyperideal of R.

Proof. We have $I \neq R$ because I is a graded 1-absorbing primary hyperideal of R. Thus λ is a non-constant graded fuzzy hyperideal of R. Suppose that $x_g^r, y_h^s, z_k^t \in FH(R)$ are homogeneous fuzzy points such that $x_g^r \circ y_h^s \circ z_k^t \subseteq \lambda$ but $x_g^r \circ y_h^s \nsubseteq \lambda$ and $z_k^t \notin FGrad(\lambda)$. Then

$$\lambda(x_g \circ y_h) = \bigwedge_{\beta \in x_g \circ y_h} \lambda(\beta) < r \wedge s$$

and

$$\lambda((z_k)^n) = \bigwedge_{\alpha \in z_k \circ \ldots \circ z_k} \lambda(\alpha) < FGrad(\lambda)(z_k) < t$$

for all $n \geq 1$. Therefore

$$\lambda(x_g \circ y_h) = \bigwedge_{\beta \in x_g \circ y_h} \lambda(\beta) = 0 \text{ and } x_g \circ y_h \nsubseteq I$$

and

$$\lambda((z_k)^n) = \bigwedge_{\alpha \in z_k \circ \cdots \circ z_k} \lambda(\alpha) = 0 \text{ and } z_k^n = \underbrace{z_k \circ \cdots \circ z_k}_{n \text{ times}} \nsubseteq I \text{ so } z_k \notin Grad(I).$$

Since I is a graded 1-absorbing primary hyperideal of R, we have $x_g \circ y_h \circ z_k \nsubseteq I$ and so $\lambda(x_g \circ y_h \circ z_k) = 0$. Also by our hypothesis, we have

$$(x_g \circ y_h \circ z_k)_{(r \wedge s \wedge t)} = x_g^r \circ y_h^s \circ z_k^t \subseteq \lambda$$

and

$$r \wedge s \wedge t \leq \lambda(x_g \circ y_h \circ z_k) = \bigwedge_{\beta \in x_g \circ y_h \circ z_k} \lambda(\beta) = 0.$$

Hence $r \wedge s = 0$ or $t = 0$, which is a contradiction. Thus

$$x_g^r \circ y_h^s \subseteq \lambda \text{ or } z_k^t \in FGrad(\lambda)$$

and λ is a graded fuzzy 1-absorbing primary hyperideal of R. □

Definition 3.21. *Let λ be a graded fuzzy 1-absorbing primary hyperideal of R. Then $\eta = FGrad(\lambda)$ is a graded prime hyperideal of R by Theorem 3.17. Hence we call λ a graded fuzzy η-1-absorbing primary hyperideal of R.*

Theorem 3.22. *Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be graded fuzzy η-1-absorbing primary hyperideals of R for some graded fuzzy 1-absorbing primary hyperideal η of R. Then $\lambda = \bigcap_{i=1}^{n} \lambda_i$ is a graded fuzzy η-1-absorbing primary hyperideal of R.*

Proof. Suppose that $x_g^r \circ y_h^s \circ z_k^t \subseteq \lambda$ and $x_g^r \circ y_h^s \nsubseteq \lambda$ where $x_g^r, y_h^s, z_k^t \in FH(R)$. Then $x_g^r \circ y_h^s \nsubseteq \lambda_i$ for some $n \geq i \geq 1$ and $x_g^r \circ y_h^s \circ z_k^t \subseteq \lambda_i$ for all $n \geq i \geq 1$. Since η_i is a graded fuzzy 1-absorbing primary hyperideal of R, we have $z_k^t \in FGrad(\lambda_i) = \eta = \bigcap_{i=1}^{n} FGrad(\lambda_i) = FGrad(\bigcap_{i=1}^{n} \lambda i) = FGrad(\lambda)$. So λ is a graded fuzzy η-1-absorbing primary hyperideal of R. □

Theorem 3.23. *Let $\{\lambda_i \mid i \in I\}$ be a directed collection of graded fuzzy 1-absorbing primary hyperideals of R. Then the graded fuzzy hyperideal $\lambda = \bigcup_{i \in I} \lambda_i$ is a graded fuzzy 1-absorbing primary hyperideal of R.*

Proof. Suppose that $x_g^r \circ y_h^s \circ z_k^t \subseteq \lambda$ and $x_g^r \circ y_h^s \nsubseteq \lambda$ for some $x_g^r, y_h^s, z_k^t \in FH(R)$. Then there exists $j \in I$ such that $x_g^r \circ y_h^s \circ z_k^t \subseteq \lambda_j$ and $x_g^r \circ y_h^s \nsubseteq \lambda_j$ for all $j \in I$. Since λ_j is a graded fuzzy 1-absorbing primary hyperideal of R, then we have

$$z_k^t \in FGrad(\lambda_j) \subseteq \bigcup_{i \in I} FGrad(\lambda_i) = FGrad(\bigcup_{i \in I} \lambda i) = FGrad(\lambda).$$

Thus $\lambda = \bigcup_{i \in I} \lambda_i$ is a graded fuzzy 1-absorbing primary hyperideal of R. □

Definition 3.24. *[21] Let $R = \bigoplus_{g \in G} R_g$ and $S = \bigoplus_{g \in G} S_g$ be two G-graded multiplicative hyperrings. The function $f : R \to S$ is called a graded homomorphism, if*

(i) for any $a, b \in R$, $f(a + b) = f(a) + f(b)$,

(ii) for any $a, b \in R$, $f(a \circ b) \subseteq f(a) \circ f(b)$,

(iii) $f(R_g) \subseteq S_g$ for any $g \in G$.

In particular, f is called a graded good homomorphism in case $f(a \circ b) = f(a) \circ f(b)$. The kernel of a graded homomorphism is defined as

$$Ker(f) = f^{-1}(\langle 0 \rangle) = \{r \in R : f(r) \in \langle 0 \rangle\}$$

and note that $f(0)$ may not be a zero element.

If Q is a graded hyperideal of S and $f : R \to S$ is a graded good homomorphism, then $f^{-1}(Q)$ is a graded hyperideal of R. If I is a graded hyperideal of R and $f : R \to S$ is an onto graded good homomorphism, then $f(I)$ is a graded hyperideal of S.

Theorem 3.25. *Let $f : R \to S$ be an onto graded good homomorphism of graded multiplicative hyperrings. If λ is a graded fuzzy 1-absorbing primary hyperideal of R, which is constant on Kerf, then $f(\lambda)$ is a graded fuzzy 1-absorbing primary hyperideal of S.*

Proof. Assume that $x_g^r \circ y_h^s \circ z_k^t \subseteq f(\lambda)$, where x_g^r, y_h^s, z_k^t are homogeneous fuzzy points of S where $g, h, k \in G$ and $r, s, t \in [0, 1]$. Since f is an onto graded good homomorphism, then there exist $a_{g_1}, b_{g_2}, c_{g_3} \in H(R)$ such that $f(a_{g_1}) = x_g, f(b_{g_2}) = y_h, f(c_{g_3}) = z_k$. Thus from

$$(x_g \circ y_h \circ z_k)_{r \wedge s \wedge t} = x_g^r \circ y_h^s \circ z_k^t \subseteq f(\lambda)$$

we have

$$\begin{aligned} r \wedge s \wedge t \leq f(\lambda)(x_g \circ y_h \circ z_k) &= \bigwedge_{\beta \in x_g \circ y_h \circ z_k} f(\lambda)(\beta) \\ &= f(\lambda)(f(a_{g_1}) \circ f(b_{g_2}) \circ f(c_{g_3})) = \bigwedge_{\theta \in f(a_{g_1}) \circ f(b_{g_2}) \circ f(c_{g_3})} f(\lambda)(\theta) \\ &= f(\lambda)(f(a_{g_1} \circ b_{g_2} \circ c_{g_3})) = \bigwedge_{\alpha \in a_{g_1} \circ b_{g_2} \circ c_{g_3}} f(\lambda)(f(\alpha)) \\ &= \lambda(a_{g_1} \circ b_{g_2} \circ c_{g_3}) = \bigwedge_{\alpha \in a_{g_1} \circ b_{g_2} \circ c_{g_3}} \lambda(\alpha) \end{aligned}$$

because λ is constant on $Kerf$. Then we get $a^r_{g_1} \circ b^s_{g_2} \circ c^t_{g_3} \subseteq \lambda$. Since λ is a graded fuzzy 1-absorbing primary hyperideal of R, then $a^r_{g_1} \circ b^s_{g_2} \subseteq \lambda$ or $c^t_{g_3} \in FGrad(\lambda)$. Thus,

$$\begin{aligned} r \wedge s \leq \lambda(a_{g_1} \circ b_{g_2}) &= \bigwedge_{\alpha' \in a_{g_1} \circ b_{g_2}} \lambda(\alpha') \\ &= f(\lambda)(f(a_{g_1} \circ b_{g_2})) = \bigwedge_{\alpha' \in a_{g_1} \circ b_{g_2}} f(\lambda)(f(\alpha')) \\ &= f(\lambda)(x_g \circ y_h) = \bigwedge_{\gamma \in x_g \circ y_h} f(\lambda)(\gamma) \end{aligned}$$

and so $x^r_g \circ y^s_h \subseteq f(\lambda)$ or

$$\begin{aligned} t &\leq FGrad(\lambda)(c_{g_3}) \\ &= f(FGrad(\lambda))(f(c_{g_3})) \\ &= f(FGrad(\lambda))(z_k) \end{aligned}$$

so $z^t_k \in f(FGrad(\lambda))$. Hence $f(\lambda)$ is a graded fuzzy 1-absorbing primary hyperideal of S. □

Theorem 3.26. *Let $f : R \to S$ be a graded good homomorphism of graded multiplicative hyperrings. If μ is a graded fuzzy 1-absorbing primary hyperideal of S, then $f^{-1}(\mu)$ is a graded fuzzy 1-absorbing primary hyperideal of R.*

Proof. Let $x^r_g \circ y^s_h \circ z^t_k \subseteq f^{-1}(\mu)$, where x^r_g, y^s_h, z^t_k are homogeneous fuzzy points of R and $g, h, k \in G$. Then

$$\begin{aligned} r \wedge s \wedge t \leq f^{-1}(\mu)(x_g \circ y_h \circ z_k) &= \bigwedge_{\beta' \in x_g \circ y_h \circ z_k} f^{-1}(\mu)(\beta') \\ &= \mu(f(x_g \circ y_h \circ z_k)) = \bigwedge_{\beta' \in x_g \circ y_h \circ z_k} \mu(f(\beta')) \\ &= \mu(f(x_g) \circ f(y_h) \circ f(z_k)) = \bigwedge_{\alpha \in f(x_g) \circ f(y_h) \circ f(z_k)} \mu(\alpha). \end{aligned}$$

Assume that $f(x_g) = a_{g_1}, f(y_h) = b_{g_2}, f(z_k) = c_{g_3} \in S$. Thus we have that

$$r \wedge s \wedge t \leq \mu(a_{g_1} \circ b_{g_3} \circ c_{g_3}) = \bigwedge_{\beta \in a_{g_1} \circ b_{g_3} \circ c_{g_3}} \mu(\beta)$$

and $a^r_{g_1} \circ b^s_{g_2} \circ c^t_{g_3} \subseteq \mu$. Since μ is a graded fuzzy 1-absorbing primary hyperideal of R, then

$$a^r_{g_1} \circ b^s_{g_2} \subseteq \mu \text{ or } c^t_{g_3} \in FGrad(\mu).$$

If $a^r_{g_1} \circ b^s_{g_2} \subseteq \mu$, then

$$\begin{aligned} r \wedge s \leq \mu(a_{g_1} \circ b_{g_2}) &= \bigwedge_{\alpha' \in a_{g_1} \circ b_{g_2}} \mu(\alpha') \\ &= \mu(f(x_g) \circ f(y_h)) = \bigwedge_{\theta' \in f(x_g) \circ f(y_h)} \mu(\theta') \\ &= \mu(f(x_g \circ y_h)) = \bigwedge_{\gamma \in x_g \circ y_h} \mu(f(\gamma)) \\ &= f^{-1}(\mu(x_g \circ y_h) = \bigwedge_{\gamma \in x_g \circ y_h} f^{-1}(\mu(\gamma)). \end{aligned}$$

Thus we get $x^r_g \circ y^s_h \subseteq f^{-1}(\mu)$. If $c^t_{g_3} \in FGrad(\mu)$, then

$$\begin{aligned} t &\leq FGrad(\mu)(c_{g_3}) \\ &= FGrad(\mu(f(z_k))) \\ &= f^{-1}(FGrad(\mu(z_k))) \end{aligned}$$

so we have $z^t_k \in f^{-1}(FGrad(\mu))$. Therefore the proof is complete. □

Definition 3.27. *Let λ be a graded fuzzy hyperideal of R. λ is said to be a graded fuzzy strongly 1-absorbing primary hyperideal of R if it is non-constant and whenever ν_1, ν_2, ν_3 are graded fuzzy hyperideals of R with $\nu_1 \circ \nu_2 \circ \nu_3 \subseteq \lambda$, then*

$$\nu_1 \circ \nu_2 \subseteq \lambda \text{ or } \nu_3 \subseteq FGrad(\lambda).$$

Theorem 3.28. *Every graded fuzzy strongly 1-absorbing primary hyperideal of R is a graded fuzzy 1-absorbing primary hyperideal of R.*

Proof. The proof is straightforward. □

Corollary 3.29. *Let λ be a graded hyperideal of a strongly distributive graded multiplicative hyrperring R. Then λ is a graded fuzzy strongly 1-absorbing primary hyperideal of R if and only if for any fuzzy subgroups $\eta_{g_1}, \eta_{g_2}, \eta_{g_3}$ of $R_{g_1}, R_{g_2}, R_{g_3}$ respectively, $\eta_{g_1} \circ \eta_{g_2} \circ \eta_{g_3} \subseteq \lambda$, then*

$$\eta_{g_1} \circ \eta_{g_2} \subseteq \lambda \text{ or } \eta_{g_3} \subseteq FGrad(\lambda)$$

where $g_1, g_2, g_3 \in G$.

4. Graded fuzzy weakly completely 1-absorbing primary hyperideals

In this section, we introduce the new concepts of graded fuzzy 1-absorbing primary hyperideals of a G-graded multiplicative hyperring R such as graded fuzzy weakly completely 1-absorbing prime hyperideals, graded fuzzy weakly completely primary hyperideals and graded fuzzy weakly completely 1-absorbing primary hyperideals. Some basic properties and characterizations of these structures are proved.

Let R be a G-graded commutative multiplicative hyperring with identity. We denote by $FH(R)$ the set of all homogeneous fuzzy points of $(R, +, \circ)$.

Definition 4.1. (a) *Let λ be a non-constant graded fuzzy hyperideal of R. Then λ is said to be a graded fuzzy weakly completely 1-absorbing prime hyperideal if for any non-unit elements $x_g, y_h, z_k \in H(R)$,*

$$\lambda(x_g \circ y_h \circ z_k) = \bigwedge_{\alpha \in x_g \circ y_h \circ z_k} \lambda(\alpha) \leq \lambda(x_g \circ y_h) = \bigwedge_{\beta \in x_g \circ y_h} \lambda(\beta)$$

or

$$\lambda(x_g \circ y_h \circ z_k) = \bigwedge_{\alpha \in x_g \circ y_h \circ z_k} \lambda(\alpha) \leq \lambda(z_k).$$

(*b*) *Let η be a non-constant graded fuzzy hyperideal of R. Then η is called a graded fuzzy weakly completely primary hyperideal of R if for all non-unit elements $x_g, y_h, z_k \in H(R)$,*

$$\lambda(x_g \circ y_h) = \bigwedge_{\alpha \in x_g \circ y_h} \lambda(\alpha) \leq \lambda(x_g)$$

or

$$\lambda(x_g \circ y_h) = \bigwedge_{\alpha \in x_g \circ y_h} \lambda(\alpha) \leq FGrad(\lambda)(y_k).$$

(*c*) *Let λ be a non-constant graded fuzzy hyperideal of R. Then λ is said to be a graded fuzzy weakly completely 1-absorbing primary hyperideal if for any non-unit elements $x_g, y_h, z_k \in H(R)$,*

$$\lambda(x_g \circ y_h \circ z_k) = \bigwedge_{\alpha \in x_g \circ y_h \circ z_k} \lambda(\alpha) \leq \lambda(x_g \circ y_h) = \bigwedge_{\beta \in x_g \circ y_h} \lambda(\beta)$$

or

$$\lambda(x_g \circ y_h \circ z_k) = \bigwedge_{\alpha \in x_g \circ y_h \circ z_k} \lambda(\alpha) \leq FGrad(\lambda)(z_k).$$

Example 4.2. *Consider $\mathbb{Z}_2$-graded multiplicative hyperring $R = (\mathbb{Z}_A[i], +, \circ)$ with $A = \{1, 2, 4\}$ and take the graded fuzzy hyperideal*

$$\zeta(x) = \begin{cases} 1 & x = 0; \\ \frac{3}{4} & x \in \langle 8 \rangle - \{0\}; \\ \frac{1}{2} & x \in \mathbb{Z}[i] - \langle 8 \rangle. \end{cases}$$

Then ζ is a graded fuzzy weakly completely 1-absorbing primary hyperideal. Assume that $\zeta(x_g \circ y_h \circ z_k) > \zeta(x_g \circ y_h)$ for any $x_g, y_h, z_k \in H(R)$. Hence $\zeta(x_g \circ y_h \circ z_k) = \frac{3}{4}$ and $\zeta(x_g \circ y_h) = \frac{1}{2}$, so $x_g \circ y_h \circ z_k \subseteq \langle 8 \rangle - \{0\}$ and $x_g \circ y_h \nsubseteq \langle 8 \rangle - \{0\}$. Since $\langle 8 \rangle$ is a graded primary hyperideal of R, then $z_k \in FGrad(\zeta)$ where

$$FGrad(\zeta)(x) = \begin{cases} 1 & x = 0; \\ \frac{3}{4} & x \in \langle 2 \rangle - \{0\}; \\ \frac{1}{2} & x \in \mathbb{Z}[i] - \langle 2 \rangle. \end{cases}$$

Theorem 4.3. *Every graded fuzzy weakly completely primary hyperideal of R is a graded fuzzy weakly completely 1-absorbing primary hyperideal of R.*

Proof. The proof is straightforward. □

Theorem 4.4. *(i) Every graded fuzzy primary hyperideal of R is a graded fuzzy weakly completely 1-absorbing primary hyperideal of R.*

(ii) Every graded fuzzy weakly completely primary hyperideal of R is a graded fuzzy weakly completely 1-absorbing primary hyperideal of R.

Proof. (*i*) Let λ be a graded fuzzy primary hyperideal of R. Assume that

$$\lambda(x_g \circ y_h \circ z_k) = \bigwedge_{\alpha \in x_g \circ y_h \circ z_k} \lambda(\alpha) > \lambda(x_g \circ y_h) = \bigwedge_{\beta \in x_g \circ y_h} \lambda(\beta)$$

for any $x_g, y_h, z_k \in H(R)$. From the definition of a graded fuzzy primary hyperideal of R, we get

$$\lambda(x_g \circ y_h \circ z_k) = \bigwedge_{\alpha \in x_g \circ y_h \circ z_k} \lambda(\alpha) \leq FGrad(\lambda)(z_k).$$

So λ is a graded fuzzy weakly completely 1-absorbing primary hyperideal of R.

(ii) The proof is straightforward. □

Theorem 4.5. *Let λ be a graded fuzzy hyperideal of R. The following statements are equivalent:*

(i) λ is a graded fuzzy weakly completely 1-absorbing primary hyperideal of R.

(ii) For every $\alpha \in [0, \lambda(0)]$, the level subset λ^α of λ is a 1-absorbing primary hyperideal of R.

Proof. (i) ⇒ (ii) Let λ be a graded fuzzy weakly completely 1-absorbing primary hyperideal of R. Suppose that $x_g, y_h, z_k \in H(R)$ and $\lambda(x_g \circ y_h \circ z_k) \subseteq \lambda^\alpha$ for some $\alpha \in [0, \lambda(0)]$. Then

$$\max\left\{\lambda(x_g \circ y_h) = \bigwedge_{\gamma \in x_g \circ y_h} \lambda(\gamma), FGrad(\lambda)(z_k)\right\} = \lambda(x_g \circ y_h \circ z_k) = \bigwedge_{\beta \in x_g \circ y_h \circ z_k} \lambda(\beta) \geq \alpha.$$

Hence

$$\lambda(x_g \circ y_h) = \bigwedge_{\gamma \in x_g \circ y_h} \lambda(\gamma) \geq \alpha \text{ or } FGrad(\lambda)(z_k) \geq \alpha,$$

which implies

$$x_g \circ y_h \subseteq \lambda^\alpha \text{ or } z_k \in FGrad(\lambda^\alpha).$$

Therefore, λ^α is a 1-absorbing primary hyperideal of R.

(ii) ⇒ (i) Suppose that λ^α is a 1-absorbing primary hyperideal of R for every $\alpha \in [0, \lambda(0)]$. Let

$$\lambda(x_g \circ y_h \circ z_k) = \bigwedge_{\beta \in x_g \circ y_h \circ z_k} \lambda(\beta) = \alpha$$

for any $x_g, y_h, z_k \in H(R)$. Then $x_g \circ y_h \circ z_k \subseteq \lambda^\alpha$ and λ^α is a 1-absorbing primary hyperideal. Thus it gives

$$x_g \circ y_h \subseteq \lambda^\alpha \text{ or } z_k \in FGrad(\lambda^\alpha).$$

Thus

$$\lambda(x_g \circ y_h) = \bigwedge_{\gamma \in x_g \circ y_h} \lambda(\beta) \geq \alpha \text{ or } FGrad(\lambda)(z_k) \geq \alpha,$$

which implies

$$\max\left\{\lambda(x_g \circ y_h) = \bigwedge_{\gamma \in x_g \circ y_h} \lambda(\gamma), FGrad(\lambda)(z_k)\right\} \geq \alpha = \bigwedge_{\beta \in x_g \circ y_h \circ z_k} \lambda(\beta) = \lambda(x_g \circ y_h \circ z_k).$$

Also since λ is a graded fuzzy hyperideal of R, we have

$$\lambda(x_g \circ y_h \circ z_k) = \bigwedge_{\beta \in x_g \circ y_h \circ z_k} \lambda(\beta) \geq \max\left\{\lambda(x_g \circ y_h) = \bigwedge_{\gamma \in x_g \circ y_h} \lambda(\gamma), FGrad(\lambda)(z_k)\right\}.$$

Hence

$$\lambda(x_g \circ y_h \circ z_k) = \bigwedge_{\beta \in x_g \circ y_h \circ z_k} \lambda(\beta) = \max\left\{\lambda(x_g \circ y_h) = \bigwedge_{\gamma \in x_g \circ y_h} \lambda(\gamma), FGrad(\lambda)(z_k)\right\}.$$

Therefore, λ is a graded fuzzy weakly completely 1-absorbing primary hyperideal of R. □

Theorem 4.6. *Let $f : R \to S$ be an onto graded good homomorphism of graded multiplicative hyperrings. If λ is a graded fuzzy weakly completely 1-absorbing primary hyperideal of R, which is constant on $Kerf$, then $f(\lambda)$ is a graded fuzzy weakly completely 1-absorbing primary hyperideal of S.*

Proof. Suppose that

$$f(\lambda)(x_g \circ y_h \circ z_k) = \bigwedge_{\beta \in x_g \circ y_h \circ z_k} f(\lambda)(\beta) > f(\lambda)(x_g \circ y_h) = \bigwedge_{\gamma \in x_g \circ y_h} f(\lambda)(\gamma)$$

for any $x_g, y_h, z_k \in H(R)$. Since f is an onto graded good homomorphism, then

$$f(a_{g_1}) = x_g, f(b_{g_2}) = y_h, f(c_{g_3}) = z_k$$

for some $a_{g_1}, b_{g_2}, c_{g_3} \in H(R)$. So

$$\begin{aligned}
f(\lambda)(x_g \circ y_h \circ z_k) &= \bigwedge_{\beta \in x_g \circ y_h \circ z_k} f(\lambda)(\beta) \\
&= f(\lambda)(f(a_{g_1}) \circ f(b_{g_2}) \circ f(c_{g_3})) = \bigwedge_{\theta \in f(a_{g_1}) \circ f(b_{g_2}) \circ f(c_{g_3})} f(\lambda)(\theta) \\
&\neq f(\lambda)(x_g \circ y_h) = \bigwedge_{\gamma \in x_g \circ y_h} f(\lambda)(\gamma) \\
&= f(\lambda)(f(a_{g_1}) \circ f(b_{g_2})) = \bigwedge_{\zeta \in f(a_{g_1}) \circ f(b_{g_2})} f(\lambda)(\zeta) \\
&= f(\lambda)(f(a_{g_1} \circ b_{g_2})) = \bigwedge_{\alpha \in a_{g_1} \circ b_{g_2}} f(\lambda)(f(\alpha)).
\end{aligned}$$

Since λ is constant on $Kerf$,

$$f(\lambda)(f(a_{g_1} \circ b_{g_2} \circ c_{g_3})) = \bigwedge_{\delta \in a_{g_1} \circ b_{g_2} \circ c_{g_3}} f(\lambda)(f(\delta)) = \lambda(a_{g_1} \circ b_{g_2} \circ c_{g_3}) = \bigwedge_{\delta \in a_{g_1} \circ b_{g_2} \circ c_{g_3}} \lambda(\delta)$$

and

$$f(\lambda)(f(a_{g_1} \circ b_{g_2})) = \bigwedge_{\alpha \in a_{g_1} \circ b_{g_2}} f(\lambda)(f(\alpha)) = \lambda(a_{g_1} \circ b_{g_2}) = \bigwedge_{\alpha \in a_{g_1} \circ b_{g_2}} \lambda(\alpha).$$

It means that

$$f(\lambda)(f(a_{g_1} \circ b_{g_2} \circ c_{g_3})) = \lambda(a_{g_1} \circ b_{g_2} \circ c_{g_3}) > \lambda(a_{g_1} \circ b_{g_2}) = f(\lambda)(f(a_{g_1} \circ b_{g_2})).$$

Since λ is a graded fuzzy weakly completely 1-absorbing primary hyperideal of R, we have

$$\begin{aligned}
\lambda(a_{g_1} \circ b_{g_2} \circ c_{g_3}) &= \bigwedge_{\delta \in a_{g_1} \circ b_{g_2} \circ c_{g_3}} \lambda(\delta) \\
&= f(\lambda)(f(a_{g_1}) \circ f(b_{g_2}) \circ f(c_{g_3})) = \bigwedge_{\theta \in f(a_{g_1}) \circ f(b_{g_2}) \circ f(c_{g_3})} f(\lambda)(\theta) \\
&= f(\lambda)(x_g \circ y_h \circ z_k) = \bigwedge_{\beta \in x_g \circ y_h \circ z_k} f(\lambda)(\beta) \\
&\leq FGrad(\lambda)(c_{g_3}) = f(FGrad(\lambda))(f(c_{g_3})) = f(FGrad(\lambda))(z_k)
\end{aligned}$$

so, we get $f(\lambda)(x_g \circ y_h \circ z_k) \leq f(FGrad(\lambda))(z_k)$. Thus $f(\lambda)$ is a graded fuzzy weakly completely 1-absorbing primary hyperideal of S. □

Theorem 4.7. *Let $f : R \to S$ be a graded good homomorphism of graded multiplicative hyperrings. If ν is a graded fuzzy weakly completely* 1*-absorbing primary hyperideal of S, then $f^{-1}(\nu)$ is a graded fuzzy weakly completely* 1*-absorbing primary hyperideal of R.*

Proof. Assume that

$$f^{-1}(\nu)(x_g \circ y_h \circ z_k) = \bigwedge_{\beta \in x_g \circ y_h \circ z_k} f^{-1}(\nu)(\beta) > f^{-1}(\nu)(x_g \circ y_h) = \bigwedge_{\gamma \in x_g \circ y_h} f^{-1}(\nu)(\gamma)$$

for any $x_g, y_h, z_k \in H(R)$. Then

$$\begin{aligned}
f^{-1}(\nu)(x_g \circ y_h \circ z_k) &= \bigwedge_{\beta\in x_g\circ y_h\circ z_k} f^{-1}(\nu)(\beta)\\
&= \nu(f(x_g \circ y_h \circ z_k)) = \bigwedge_{\beta\in x_g\circ y_h\circ z_k} \nu(f(\beta))\\
&= \nu(f(x_g) \circ f(y_h) \circ f(z_k)) = \bigwedge_{\theta'\in f(x_g)\circ f(y_h)\circ f(z_k)} f(\nu)(\theta')\\
&> f^{-1}(\nu)(x_g \circ y_h) = \bigwedge_{\alpha\in x_g\circ y_h} f^{-1}(\nu)(\alpha)\\
&= \nu(f(x_g \circ y_h)) = \bigwedge_{\alpha\in x_g\circ y_h} \nu(f(\alpha))\\
&= \nu(f(x_g) \circ f(y_h))) = \bigwedge_{\zeta'\in f(x_g)\circ f(y_h))} f(\nu)(\zeta').
\end{aligned}$$

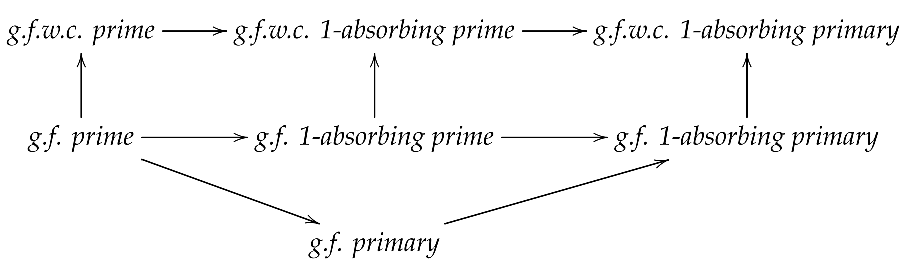
Since ν is a graded fuzzy weakly completely 1-absorbing primary hyperideal of S, we have

$$\begin{aligned}
f^{-1}(\nu)(x_g \circ y_h \circ z_k) &= \bigwedge_{\beta\in x_g\circ y_h\circ z_k} f^{-1}(\nu)(\beta)\\
&= \nu(f(x_g) \circ f(y_h) \circ f(z_k)) = \bigwedge_{\theta'\in f(x_g)\circ f(y_h)\circ f(z_k)} f(\nu)(\theta')\\
&\leq FGrad(\nu)(f(z_k)) = f^{-1}(FGrad(\nu))(z_k)\\
&= FGrad(f^{-1}(\nu)(z_k)).
\end{aligned}$$

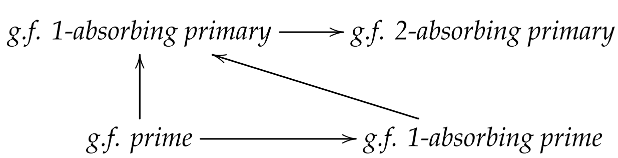
Therefore $f^{-1}(\nu)$ is a graded fuzzy weakly completely 1-absorbing primary hyperideal of R. □

Corollary 4.8. *Let f be a graded good homomorphism from R onto S. f induces a one-to-one inclusion preserving correspondence between graded fuzzy weakly completely 1-absorbing hyperideal of S in such a way that if λ is a graded fuzzy weakly completely 1-absorbing hyperideal of R constant on $Ker(f)$, then $f(\lambda)$ is the corresponding graded fuzzy weakly completely 1-absorbing hyperideal of S, and if ν is a graded fuzzy weakly completely 1-absorbing hyperideal of S, then $f^{-1}(\nu)$ is the corresponding graded fuzzy weakly completely 1-absorbing hyperideal of R.*

Remark 4.9. *Note that the following diagram shows the transition between definitions of graded fuzzy hyperideals:*



and also we have



5. Conclusions

In this article, we introduced the concepts of graded fuzzy radical of a graded fuzzy hyperideal and graded fuzzy primary hyperideals of a graded hyperring as a generalization of primary hyperideals. We investigated the basic properties of these notions and presented some examples. Furthermore, we introduced the concepts of graded fuzzy absorbing primary hyperideals of a G-graded multiplicative hyperring R such as graded fuzzy 1-absorbing primary hyperideals, graded fuzzy strongly 1-absorbing primary hyperideals, graded fuzzy 2-absorbing primary hyperideals, graded fuzzy weakly completely primary hyperideals and graded fuzzy weakly completely 1-absorbing primary hyperideals. Many basic properties and characterizations of these structures were proved. We proved that every graded fuzzy 1-absorbing primary hyperideal is a graded fuzzy 2-absorbing primary hyperideal and by an example, we showed that its converse doesn't hold. We investigated every graded fuzzy strongly 1-absorbing primary hyperideal is a graded fuzzy 1-absorbing primary hyperideal, but its converse is an open problem. Many results were given to show the relations between these new concepts and other fuzzy hyperideals. Finally, by diagrams, we showed the relationship between these definitions.

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