Filomat 38:15 (2024), 5431–5440
https://doi.org/10.2298/FIL2415431P

Published by Faculty of Sciences and Mathematics,
University of Niš, Serbia
Available at: http://www.pmf.ni.ac.rs/filomat

Every regular countably sieve-complete semitopological group is a topological group

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Abstract. In this note, we firstly discuss some properties of spaces which are countably sieve-complete, densely q-complete and strongly Baire. By some known conclusions, we finally show that if G is a regular countably sieve-complete semitopological group then G is a topological group. If a regular semitopological group G has a dense subgroup which is countably sieve-complete (densely q-complete), then G is a topological group. If G is a regular countably sieve-complete semitopological group then G is a D-space if and only if G is paracompact. We point out that some conditions in Theorem 2.14 and Corollary 2.15 in [17] are not essential.

1. Introduction

Recall that a *paratopological group* is a group with a topology such that the multiplication on the group is jointly continuous. A *topological group* G is a paratopological group such that the inverse mapping of G into itself associating x^{-1} with $x \in G$ is continuous. A *semitopological group* is a group with a topology in which the left and the right translations are continuous [4]. The set of all positive integers is denoted by $\mathbb{N}$ and $\omega = \mathbb{N} \cup \{0\}$. In notation and terminology we will follow [9].

A topological space X is called *pseudocompact* if X is a Tychonoff space and every continuous real-valued function defined on X is bounded [9]. A Tychonoff space X is pseudocompact if and only if every locally finite family of open sets in X is finite [9]. Recall that a space X is *feebly compact* if every locally finite family of open sets in X is finite.

If G is a paratopological group such that G is a dense G_δ-set in a regular feebly compact space X, then G is a topological group (([4], Theorem 2.4.1) and [3]). Thus every regular countably compact paratopological group is a topological group ([4], Corollary 2.4.4). In [20] it was proved that a completely regular countably compact semitopological group is a topological group. In [13] a completely regular pseudocompact semitopological group was constructed which is not a topological group. Applying Martin's Axiom, Ravsky constructed a Hausdorff countably compact paratopological group which is not a topological group ([19] and ([4], p. 128)).

A Tychonoff space X is *Čech-complete* if and only if X is a G_δ-set in some (equivalent, every) Hausdorff compactification of X [9]. Every Čech-complete semitopological group is a topological group (([4], Theorem

2020 *Mathematics Subject Classification*. Primary 54A20; Secondary 54A25, 54E50
Keywords. Semitopological group, countably sieve-complete, densely q-complete, countably compact
Received: 15 June 2023; Revised: 09 November 2023; Accepted: 14 November 2023
Communicated by Ljubiša D. R. Kočinac
Research supported by the National Natural Science Foundation of China (Grant No. 12171015, 62272015).
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2.4.12) and [6]). Recall that a space X is called *locally* $\mathcal{P}$ if every point x of X has a neighborhood V_x such that V_x has property $\mathcal{P}$, where $\mathcal{P}$ is topological property. Then a space X is called *locally countably compact* if every point x of X has a neighborhood V_x such that V_x is a countably compact subspace of X.

An important generalization of both Čech-complete spaces and locally countably compact spaces was introduced by Z. Frolík [10]—strongly countably complete spaces. Every G_δ-subspace and every closed subspace of a regular strongly countably complete space is a strongly countably complete space [10]. This class of spaces was used both in the study of the continuity of operations in groups ([18] and [6]), and in the study of separately continuous mappings ([16] and [11]). In [6] and [11], strongly countably complete spaces are called countably Čech-complete spaces. In [18], H. Pfister proved that every locally strongly countably complete regular paratopological group is a topological group. Consequently, every locally countably compact regular paratopological group is a topological group. In [6] A. Bouziad, using ([6], Theorem 4), proved ([6], Corollary 5): every semitopological Baire p-space group is a paratopological group. Using the same arguments, it follows from ([6], Theorem 3) that every semitopological Baire point-wise countably complete [6] completely regular group is a paratopological group. Since every strongly countably complete regular space is point-wise countably complete [6], we obtain that from the results of [6] and the mentioned result of H. Pfister [18] it follows that every semitopological strongly countably complete completely regular group is a topological group. Consequently, every semitopological locally countably compact completely regular group is a topological group.

Further generalizations of Čech-complete spaces and strongly countably complete spaces were obtained using the concept of sieve [7]. Sieve-complete spaces [15] (which are called monotonically Čech-complete in [7] and called λ_b-spaces in [24]) are a generalization of Čech-complete spaces and countably sieve-complete spaces are a generalization of strongly countably complete spaces. In [24] λ_c-spaces were introduced, which in [1] are called q-complete spaces. Any q-complete space is strongly countably complete and in the class of regular spaces these classes coincide [15]. In [1], Arhangel'skii and Choban also considered a broader class than countably sieve-complete spaces—densely q-complete spaces.

In this note, we firstly discuss some properties of spaces which are countably sieve-complete, densely q-complete and strongly Baire. By some known conclusions, we finally show that if G is a regular countably sieve-complete semitopological group then G is a topological group. If a regular semitopological group G has a dense subgroup which is countably sieve-complete (densely q-complete), then G is a topological group. If G is a regular locally countably sieve-complete semitopological group, then G is a topological group. Thus every locally countably compact regular semitopological group is a topological group. This answers Problem 2.3.B in [4].

Recall that a *neighborhood assignment* for a space X is a function ϕ from X to the topology of the space X such that $x \in \phi(x)$ for any $x \in X$ [8]. A space X is a *D-space* if for every neighborhood assignment ϕ for X there is a closed discrete subspace D of X such that $X = \bigcup\{\phi(d) : d \in D\}$ [8]. It is an open problem that whether every paracompact Hausdorff space a D-space. We point out that if G is a regular countably sieve-complete semitopological group, then G is a D-space if and only if G is paracompact. We also point out that some conditions in Theorem 2.14 and Corollary 2.15 in [17] are not essential.

2. Countably sieve-complete, densely q-complete and strongly Baire spaces

Recall that a filter base $\mathcal{F}$ *clusters at* x in X if $x \in \overline{F}$ for all $F \in \mathcal{F}$. Two collections of sets $\mathcal{F}$ and $\mathcal{U}$ *mesh* if every $F \in \mathcal{F}$ intersects every $U \in \mathcal{U}$ ([14], p. 99). A *sieve* on a space X is a sequence of open covers $\{U_\alpha : \alpha \in A_n\}_{n\in\omega}$ (with disjoint A_n), together with functions $\pi_n : A_{n+1} \to A_n$, such that, for all $n \in \omega$ and $\alpha \in A_n$, $U_\alpha = \bigcup\{U_\beta : \beta \in \pi_n^{-1}(\alpha)\}$. A *$\pi$-chain* for such a sieve is a sequence (α_n) such that $\alpha_n \in A_n$ and $\pi_n(\alpha_{n+1}) = \alpha_n$ for all n. The sieve is *complete* if, for every π-chain (α_n), every filter base $\mathcal{F}$ on X which meshes with $\{U_{\alpha_n} : n \in \omega\}$ clusters in X [15]. A space X with a complete sieve is called *sieve-complete* [15].

Recall that a point x of a space X is an *accumulation point* of a sequence $\{x_n\}_{n\in\omega}$ of points of X if every open neighborhood V of x, $|\{n \in \omega : x_n \in V\}| = \omega$. An accumulation point of a sequence $\{x_n\}$ of points of a space X is also called *cluster point* of the sequence $\{x_n\}_{n\in\omega}$. Analogously to a complete sieve, one can define a *countably complete sieve* ([15], p. 729) by restricting the filter base $\mathcal{F}$ in the definition of complete sieve to be countable (equivalently, by requiring that, if (α_n) is a π-chain and $x_n \in U_{\alpha_n}$ for all n, then the sequence $\{x_n\}_{n\in\omega}$ clusters in X). A space with a countably complete sieve is called *countably sieve-complete* ([15], p. 729). Every Čech-complete space is sieve-complete and every sieve-complete space is countably sieve-complete. Every countably compact space is countably sieve-complete, but not necessarily sieve-complete ([15], p. 730). Every uncountable discrete space X is countably sieve-complete, but it is not feebly compact.

A sequence $\{U_n : n \in \omega\}$ of open subsets of a space X is called a *stable sequence* [1] if it satisfies the following conditions:

(S1) $\emptyset \neq U_{n+1} \subset U_n$ for any $n \in \omega$;
(S2) Every sequence $\{V_n : n \in \omega\}$ of open non-empty sets in X such that $V_n \subset U_n$ for each $n \in \omega$ has an accumulation point in X.

The following notions appear in [1]. Let Y be a dense subspace of a space X, $\gamma = \{\gamma_n = \{U_\alpha : \alpha \in A_n\} : n \in \omega\}$ be a sequence of families of open subsets of X, and let $\pi = \{\pi_n : A_{n+1} \to A_n : n \in \omega\}$ be a sequence of mappings. A sequence $\alpha = \{\alpha_n : n \in \omega\}$ is called a *c-sequence* if $\alpha_n \in A_n$ and $\pi_n(\alpha_{n+1}) = \alpha_n$ for every n. Let $H(\alpha) = \bigcap\{U_{\alpha_n} : n \in \omega\}$. Consider the following conditions:

(SC1) $\bigcup\{U_\beta : \beta \in A_n\}$ is dense subset of X for every $n \in \omega$;
(SC2) $\bigcup\{U_\beta : \beta \in \pi_n^{-1}(\alpha)\}$ is a dense subset of the set U_α for all $\alpha \in A_n$ and $n \in \omega$;
(SC3) $U_\alpha = \bigcup\{U_\beta : \beta \in \pi_n^{-1}(\alpha)\}$ for all $\alpha \in A_n$ and $n \in \omega$;
(SC4) $\bigcup\{\overline{U_\beta} : \beta \in \pi_n^{-1}(\alpha)\} \subset U_\alpha$ for all $\alpha \in A_n$ and $n \in \omega$;
(C1) For any c-sequence $\alpha = \{\alpha_n \in A_n : n \in \omega\}$, the sequence $\{U_{\alpha_n} : n \in \omega\}$ is stable.
(C2) For any c-sequence $\alpha = \{\alpha_n \in A_n : n \in \omega\}$, each sequence $\{y_n \in Y \cap U_{\alpha_n} : n \in \omega\}$ has an accumulation point in X;
(C4) For any c-sequence $\alpha = \{\alpha_n \in A_n : n \in \omega\}$, the set $H(\alpha)$ is a non-empty compact subset of X;
(C5) For any c-sequence $\alpha = \{\alpha_n \in A_n : n \in \omega\}$, the set $H(\alpha)$ is a non-empty countably compact subset of X.

Sequences γ and π are called an *A-sieve* if they have the Properties (SC3) and (SC4) and each γ_n covers X. A space is called a *q-complete* if there exists an A-sieve with the Properties (C2) and (C5) for $Y = X$. A space X is called *fan-complete* if there exists an A-sieve on X with the Property (C1). Sequences γ and π are called a *dense A-sieve* if they have the Properties (SC1), (SC2), (SC4). A space is called *densely sieve-complete* if there exist a dense subspace Y and a dense A-sieve with the Properties (C2) and (C4). A space X is called *densely q-complete* if there exists a dense subspace Y and a dense A-sieve with the Property (C2). A space X is called *densely fan-complete* if there exists a dense A-sieve on X with the Property (C1).

Proposition 2.1. *(*[1], p. 37*) Any closed subspace of a q-complete space is q-complete.*

Proposition 2.2. *(*[1], p. 37*) Any q-complete space is densely q-complete.*

By definitions of countably sieve-completeness and q-completeness, we have the following result.

Proposition 2.3. *Every q-complete space is countably sieve-complete.*

A sieve $(\{U_\alpha : \alpha \in A_n\}, \pi_n)$ on a space X is a *strong sieve* if $\overline{U_\beta} \subset U_\alpha$ whenever $\alpha \in A_n$ and $\beta \in \pi_n^{-1}(\alpha)$ [7].

Lemma 2.4. *(*[15], p. 729*) Every regular countably sieve-complete space has a strong countably complete sieve.*

Proposition 2.5. *If X is a regular space, then X is countably sieve-complete if and only if X is q-complete.*

Proof. The sufficiency follows from Proposition 2.3. Now we prove the necessity. Since X is a regular countably sieve-complete space, it follows from Lemma 2.4 that X has a strong countably complete sieve $(\{U_\alpha : \alpha \in A_n\}, \pi_n)$. Then for any π-chain (α_n) and any sequence $\{y_n\}_{n\in\omega}$ with $y_n \in U_{\alpha_n}$ for every $n \in \omega$, the sequence $\{y_n\}_{n\in\omega}$ has an accumulation point y in X. Since $\overline{U_{\alpha_{n+1}}} \subset U_{\alpha_n}$ for every n, the set $H = \bigcap\{U_{\alpha_n} : n \in \omega\}$ is a closed non-empty countably compact subset of X. Thus X is a q-complete space. □

By Propositions 2.2 and 2.5, we have the following result.

Proposition 2.6. *Every regular countably sieve-complete space is densely q-complete.*

In what follows, we show that the converse of the above result does not hold.

Recall that $\mathbb{R}$ is the set of real numbers. The Michael line M is the set $\mathbb{R}$ topologized by isolating the points of the set $\mathbb{P}$ of irrational numbers and leaving the points of the set $\mathbb{Q}$ of rational numbers with their usual neighborhoods. The following result shows that the Michael line M is densely q-complete.

Theorem 2.7. *Let X be a regular space and let Y be a dense subspace of X. If Y is densely q-complete, then so is X.*

Proof. By assumption, there exist a dense subspace D of Y and a dense A-sieve $\mathcal{U} = \{\gamma_n = \{U_\alpha : \alpha \in A_n\}, \pi_n : A_{n+1} \to A_n : n \in \omega\}$ with the Property (C2). Since D is dense in Y and Y is dense in X, the set D is dense in X. Now we define a dense A-sieve $\mathcal{V} = \{\mathcal{V}_n = \{V_{\langle\alpha,\lambda\rangle} : \langle\alpha,\lambda\rangle \in A_n\times\Lambda_n\}, \pi_n\times\phi_n : A_{n+1}\times\Lambda_{n+1} \to A_n\times\Lambda_n : n \in \omega\}$ on X such that the set D and the dense A-sieve $\mathcal{V}$ on X satisfy the Property (C2), where $\phi_n : \Lambda_{n+1} \to \Lambda_n$ is a mapping. The dense A-sieve $\mathcal{V}$ on X also has the following properties:

1. If $\langle\alpha,\lambda\rangle \in A_0 \times \Lambda_0$, then $V_{\langle\alpha,\lambda\rangle}$ is an open subset of X such that $V_{\langle\alpha,\lambda\rangle} \cap Y = U_\alpha$, where Λ_0 is any non-empty set;
2. For any $n \in \omega$ and any $\langle\alpha,\lambda\rangle \in A_n \times \Lambda_n$, the set $V_{\langle\alpha,\lambda\rangle}$ is an open subset of X such that $V_{\langle\alpha,\lambda\rangle} \cap Y \subset U_\alpha$ and if $\langle\beta,\lambda'\rangle \in \pi_n^{-1}(\alpha)\times\phi_n^{-1}(\lambda)$ then $\overline{V_{\langle\beta,\lambda'\rangle}} \subset V_{\langle\alpha,\lambda\rangle}$ and $V_{\langle\beta,\lambda'\rangle} \cap Y \subset U_\beta$.

Let Λ_0 be any non-empty set. For for any $\alpha \in A_0$ and any $\lambda \in \Lambda_0$, let $V_{\langle\alpha,\lambda\rangle}$ be an open subset of X such that $V_{\langle\alpha,\lambda\rangle} \cap Y = U_\alpha$. Denote $\mathcal{T}' = \{O \subset X : O$ is a non-empty open subset of $X\}$ and let $\kappa = |\mathcal{T}'|$. Then denote $\mathcal{T}' = \{O_\xi : \xi \in \kappa\}$. Let $\Lambda_1 = \Lambda_0 \times \kappa$ and let $\phi_0 : \Lambda_1 \to \Lambda_0$ be a mapping such that $\phi_0(\langle\lambda,\xi\rangle) = \lambda$ for every $\langle\lambda,\xi\rangle \in \Lambda_1$. If $\langle\alpha,\lambda\rangle \in A_0\times\Lambda_0$ and $\langle\beta,\lambda'\rangle \in \pi_0^{-1}(\alpha)\times\phi_0^{-1}(\lambda)$, then we let $V_{\langle\beta,\lambda'\rangle}$ be an open subset of X such that $V_{\langle\beta,\lambda'\rangle} = O_{\lambda'}$ if $O_{\lambda'} \cap Y \subset U_\beta$ and $\overline{O_{\lambda'}} \subset V_{\langle\alpha,\lambda\rangle}$, otherwise $V_{\langle\beta,\lambda'\rangle} = \emptyset$.

Let $n \in \omega$. Assume that for every $i < n$ we have defined a mapping $\phi_i : \Lambda_{i+1} \to \Lambda_i$ and for every $i \le n$ and any $\langle\alpha,\lambda\rangle \in A_i \times \Lambda_i$ there exists an open subset $V_{\langle\alpha,\lambda\rangle}$ of X with the following properties:

1. $V_{\langle\alpha,\lambda\rangle} \cap Y = U_\alpha$ if $\langle\alpha,\lambda\rangle \in A_0\times\Lambda_0$;
2. If $0 \le i < n$ and $\langle\alpha,\lambda\rangle \in A_i\times\Lambda_i$, then for any $\langle\beta,\lambda'\rangle \in \pi_i^{-1}(\alpha)\times\phi_i^{-1}(\lambda)$, $V_{\langle\beta,\lambda'\rangle}$ is an open subset of X such that $V_{\langle\beta,\lambda'\rangle} \cap Y \subset U_\beta$ and $\overline{V_{\langle\beta,\lambda'\rangle}} \subset V_{\langle\alpha,\lambda\rangle}$;
3. If $0 \le i < n$ and $\langle\alpha,\lambda\rangle \in A_i\times\Lambda_i$, then $\bigcup\{V_{\langle\beta,\lambda'\rangle} : \langle\beta,\lambda'\rangle \in \pi_i^{-1}(\alpha)\times\phi_i^{-1}(\lambda)\}$ is dense in $V_{\langle\alpha,\lambda\rangle}$.

Let $\Lambda_{n+1} = \Lambda_n\times\kappa$ and let $\phi_n : \Lambda_{n+1} \to \Lambda_n$ be a mapping such that $\phi_n(\langle\lambda,\lambda'\rangle) = \lambda$ whenever $\langle\lambda,\lambda'\rangle \in \Lambda_{n+1}$.

For any $\langle\alpha,\lambda\rangle \in A_n\times\Lambda_n$ and any $\langle\beta,\lambda'\rangle \in \pi_n^{-1}(A_n)\times\phi_n^{-1}(\Lambda_n)$, let $V_{\langle\beta,\lambda'\rangle} = O_{\lambda'}$ if $O_{\lambda'} \cap Y \subset U_\beta$ and $\overline{O_{\lambda'}} \subset V_{\langle\alpha,\lambda\rangle}$, otherwise $V_{\langle\beta,\lambda'\rangle} = \emptyset$. Now we assume that $V_{\langle\alpha,\lambda\rangle} \neq \emptyset$. Since Y is dense in X and $V_{\langle\alpha,\lambda\rangle}$ is a non-empty open subset of X, $Y \cap V_{\langle\alpha,\lambda\rangle}$ is dense in $V_{\langle\alpha,\lambda\rangle}$. Since $V_{\langle\alpha,\lambda\rangle} \cap Y \subset U_\alpha$ and $\bigcup\{U_\beta : \beta \in \pi_n^{-1}(\alpha)\}$ is dense in U_α, the set $V_{\langle\alpha,\lambda\rangle} \cap (\bigcup\{U_\beta : \beta \in \pi_n^{-1}(\alpha)\})$ is dense in $V_{\langle\alpha,\lambda\rangle}$. Thus $\bigcup\{O_\xi : \xi \in \kappa, O_\xi \cap Y \subset U_\beta$ and $\overline{O_\xi} \subset V_{\langle\alpha,\lambda\rangle}\}$ is dense in $V_{\langle\alpha,\lambda\rangle}$. Then the set $\bigcup\{V_{\langle\beta,\lambda'\rangle} : \langle\beta,\lambda'\rangle \in \pi_n^{-1}(\alpha)\times\phi_n^{-1}(\lambda)\}$ is dense in $V_{\langle\alpha,\lambda\rangle}$.

In this way, we get a dense A-sieve $\mathcal{V} = \{\mathcal{V}_n = \{V_{\langle\alpha,\lambda\rangle} : \langle\alpha,\lambda\rangle \in A_n\times\Lambda_n\} : \pi_n\times\phi_n : A_{n+1}\times\Lambda_{n+1} \to A_n\times\Lambda_n : n \in \omega\}$ on X. If $\{\langle\alpha_n,\lambda_n\rangle : n \in \omega\}$ is a c-sequence, then $V_{\langle\alpha_n,\lambda_n\rangle} \cap Y \subset U_{\alpha_n}$, $\pi_n(\alpha_{n+1}) = \alpha_n$ and $\overline{V_{\langle\alpha_{n+1},\lambda_{n+1}\rangle}} \subset V_{\langle\alpha_n,\lambda_n\rangle}$ for every $n \in \omega$.

If $\{d_n\}_{n\in\omega}$ is a sequence of points such that $d_n \in V_{\langle\alpha_n,\lambda_n\rangle} \cap D$ for every $n \in \omega$, then $d_n \in U_{\alpha_n}$ for every $n \in \omega$. Since $\mathcal{U}$ is a dense A-sieve on Y such that D and $\mathcal{U}$ satisfy the Property (C2), the sequence $\{d_n\}_{n\in\omega}$ has an accumulation in Y. Then the sequence $\{d_n\}_{n\in\omega}$ has an accumulation in X. Thus $\mathcal{V}$ is a dense A-sieve on X. Then the dense subspace D of X and the dense A-sieve $\mathcal{V}$ on X satisfy the Property (C2). Thus X is densely q-complete. □

The following result was proved in ([15], Theorem 3.2). A paracompact Hausdorff space X is Čech-complete if and only if X is sieve-complete. In ([15], p. 730), it was pointed out that the above result valid with "sieve-complete" weakened to "countably sieve-complete".

Lemma 2.8. ([15], p. 730) *A paracompact Hausdorff space X is Čech-complete if and only if X is countably sieve-complete.*

Remark 2.9. The Michael line M is densely q-complete, but it is not q-complete (countably sieve-complete).

Proof. The space $\mathbb{Q}$ of all rational numbers with the topology of a subspace of the real line with the usual topology is not Čech-complete ([9], p. 200). Thus the subspace $\mathbb{Q}$ of M is not Čech-complete. Since every closed subspace of a Čech-complete space is Čech-complete ([9], Theorem 3.9.6), the Michael line M is not Čech-complete. By Lemma 2.8, M is not countably sieve-complete. By Proposition 2.5, M is not q-complete.

Since the subspace $\mathbb{P}$ of M is a densely q-complete dense subspace of X, it follows from Theorem 2.7 M is densely q-complete. □

By Proposition 2.6 and Theorem 2.7, we have the following result.

Proposition 2.10. *Let X be a regular space and let Y be a dense subspace of X. If Y is countably sieve-complete, then X is a densely q-complete space.*

By an argument similar to the proof of Theorem 2.7, we have the following result.

Proposition 2.11. ([1], p. 38) *If X is a regular space and Y is a dense subspace of X such that Y is densely fan-complete, then X is densely fan-complete.*

Proposition 2.12. ([15], p. 729) *Countably sieve-completeness is inherited by closed subsets.*

Recall that a subset F of a space X is called a *regular closed set* if $F = \overline{F^\circ}$.

Proposition 2.13. *Let X be a regular space and let Y be a regular closed subset of X. If X is densely q-complete (densely sieve-complete, densely fan-complete, fan-complete, q-complete), then the subspace Y of X is densely q-complete (densely sieve-complete, densely fan-complete, fan-complete, q-complete).*

Proof. We just prove the case of densely q-completeness. The proofs of other cases are similar. Since X is densely q-complete, there exist a dense subspace D of X and a dense A-sieve $\mathcal{U} = \{\mathcal{U}_\alpha = \{U_\alpha : \alpha \in A_n\}, \pi_n : A_{n+1} \to A_n : n \in \omega\}$ with the Property (C2). Since Y is a regular closed set, $Y = \overline{Y^\circ}$. Then $D_Y = D \cap Y^\circ$ is dense in Y. If $\mathcal{U}_Y = \{\mathcal{U}'_\alpha = \{U_\alpha \cap Y : \alpha \in A_n\}, \pi_n : A_{n+1} \to A_n : n \in \omega\}$, then $\mathcal{U}_Y$ is a dense A-sieve on Y. It is obvious that the dense subset D_Y of Y and the dense A-sieve $\mathcal{U}_Y$ satisfy the Property (C2). □

Proposition 2.14. *Let X be a regular space and let Y be an open subspace of X. If X is densely q-complete (densely sieve-complete, densely fan-complete), then so is Y.*

Proof. We just prove the case of densely q-completeness. The proofs of other cases are similar.

By assumption, there exist a dense subspace D of X and a dense A-sieve $\mathcal{U} = \{\gamma_n = \{U_\alpha : \alpha \in A_n\}, \pi_n : A_{n+1} \to A_n : n \in \omega\}$ with the Property (C2). Since Y is open in X and $\overline{D} = X$, $D_Y = D \cap Y$ is dense in Y. Denote $\mathcal{T}' = \{O \subset X : O$ is a non-empty open subset of $X\}$ and let $\kappa = |\mathcal{T}'|$. Then denote $\mathcal{T}' = \{O_\xi : \xi \in \kappa\}$.

By an argument similar to the proof of Theorem 2.7 we can get a dense A-sieve $\mathcal{V} = \{\mathcal{V}_n = \{V_{\langle\alpha,\lambda\rangle} : \langle\alpha,\lambda\rangle \in A_n \times \Lambda_n\}, \pi_n \times \phi_n : A_{n+1} \times \Lambda_{n+1} \to A_n \times \Lambda_n : n \in \omega\}$ on X such that D_Y and $\mathcal{V}$ satisfy the Property (C2) and the following properties:

1. Λ_0 is any non-empty set and $V_{\langle\alpha,\lambda\rangle} = U_\alpha \cap Y$ for every $\langle\alpha,\lambda\rangle \in A_0 \times \Lambda_0$;
2. For every $n \in \omega$, let $\Lambda_{n+1} = \Lambda_n \times \kappa$ and let $\phi_n : \Lambda_{n+1} \to \Lambda_n$ be a mapping such that $\phi_n(\langle\lambda,\lambda'\rangle) = \lambda$ whenever $\langle\lambda,\lambda'\rangle \in \Lambda_{n+1}$.
3. For any $n \in \omega$ and any $\langle\alpha,\lambda\rangle \in A_n \times \Lambda_n$, the set $V_{\langle\beta,\lambda'\rangle}$ is an open subset of X such that $\overline{V_{\langle\beta,\lambda'\rangle}} \subset U_\beta \cap V_{\langle\alpha,\lambda\rangle} \cap Y$ for any $\langle\beta,\lambda'\rangle \in \pi_n^{-1}(\alpha) \times \phi_n^{-1}(\lambda)$;
4. For any $n \in \omega$ and any $\langle\alpha,\lambda\rangle \in A_n \times \Lambda_n$, the set $\bigcup\{V_{\langle\beta,\lambda'\rangle} : \langle\beta,\lambda'\rangle \in \pi_i^{-1}(\alpha) \times \phi_i^{-1}(\lambda)\}$ is dense in $V_{\langle\alpha,\lambda\rangle}$.

If $\{\langle \alpha_n, \lambda_n \rangle : n \in \omega\}$ is any c-sequence, then $\pi_n(\alpha_{n+1}) = \alpha_n$ and $\phi_n(\lambda_{n+1}) = \lambda_n$ for every $n \in \omega$. If $d_n \in D_Y \cap V_{\langle \alpha_n, \lambda_n \rangle}$ for every $n \in \omega$, then $d_n \in U_{\alpha_n}$ for every $n \in \omega$. Thus the sequence $\{d_n\}_{n\in\omega}$ has an accumulation point $y \in X$. Since $V_{\langle \alpha_0, \lambda_0 \rangle} \subset Y$ and $V_{\langle \alpha_n, \lambda_n \rangle} \subset V_{\langle \alpha_1, \lambda_1 \rangle} \subset \overline{V_{\langle \alpha_1, \lambda_1 \rangle}} \subset Y$, the point $y \in Y$. Thus Y is densely q-complete. □

Proposition 2.15. ([1], Proposition 2.3) *Every G_δ-subspace of a regular fan-complete space is fan-complete.*

Proposition 2.16. *Every G_δ-subspace of a regular q-complete space is q-complete.*

Proof. This can be gotten by Proposition 2.5 and the fact that countably sieve-completeness is inherited by G_δ-subsets in a regular space ([15], p. 729). □

It was pointed out in ([15], p. 729) that a space X is countably sieve-complete if and only if every point of X has a countably sieve-complete open neighborhood. If X is regular, then the neighborhood need not be open. However, we have following result.

Lemma 2.17. *A space X is countably sieve-complete if and only if every point of X has a countably sieve-complete neighborhood.*

Proof. The necessity is obvious. Now we prove the sufficiency.

For every $x \in X$, there exists a countably sieve-complete neighborhood V_x. Let $\mathcal{U}_x = \{\mathcal{U}_n(x) = \{U_\alpha : \alpha \in A_n(x)\}, \pi_{n,x} : A_{n+1}(x) \to A_n(x), n \in \omega\}$ be a countably complete sieve on X. For every $x \in X$ and every $n \in \omega$ we let $\mathcal{U}'_n(x) = \{U_\alpha \cap V_x^\circ : \alpha \in A_n(x)\}$. For every $n \in \omega$, let $B_n = \bigcup\{A_n(x) \times \{x\} : x \in X\}$ and let $\pi_n : B_{n+1} \to B_n$ be a mapping such that for any $x \in X$ $\pi_n(\langle \alpha, x \rangle) = \alpha$ whenever $\langle \alpha, x \rangle \in A_n(x) \times \{x\}$. For any $x \in X$ and any $\langle \alpha, x \rangle \in A_n(x) \times \{x\}$, let $V_{\langle \alpha, x \rangle} = U_\alpha \cap V_x^\circ$. Then $\mathcal{V} = \{\mathcal{V}_n = \{V_{\langle \alpha, x \rangle} : \langle \alpha, x \rangle \in B_n\}, \pi_n : B_{n+1} \to B_n : n \in \omega\}$ is a sieve on X. If $(\langle \alpha_n, x_n \rangle)$ is a π-chain, then there exists $y \in X$ such that $x_n = y$ and $\alpha_n \in A_n(y)$ for every $n \in \omega$. If $\{d_n\}_{n\in\omega}$ is a sequence of points of X such that $d_n \in V_{\langle \alpha_n, x_n \rangle}$ for every $n \in \omega$, then $d_n \in U_{\alpha_n} \cap V_y^\circ \subset U_{\alpha_n}$ and $\pi_{n,y}(\alpha_{n+1}) = \alpha_n$ for every $n \in \omega$.

Since $\mathcal{U}_y$ is a countably complete sieve on V_y, the sequence $\{d_n\}_{n\in\omega}$ has an accumulation point d in $V_y \subset X$. Thus $\mathcal{V}$ is a countably complete sieve on X. Then X is countably sieve-complete. □

By Lemma 2.17 and Proposition 2.5, we have the following result.

Proposition 2.18. *A regular space X is q-complete if and only if every point x of X has a q-complete neighborhood.*

Proposition 2.19. *Every locally countably compact space X is countably sieve-complete.*

Proof. Since X is locally countably compact and every countably compact space is countably sieve-complete, every point of X has a neighborhood which is countably sieve-complete. Thus by Lemma 2.17 X is countably sieve-complete. □

The above result shows that the T_1 separation axiom in Proposition 1.1 in [17] is not essential.

Proposition 2.20. *Let X be a regular space. If every point of X has a densely fan-complete (fan-complete) neighborhood, then X is densely fan-complete (fan-complete).*

Proof. We just prove the case of densely fan-completeness. The proof of the other case is similar.

Since X is regular and every point of X has a densely fan-complete neighborhood, it follows from Proposition 2.13 for every $x \in X$ there exists an open neighborhood V_x of x such that $\overline{V_x}$ is densely fan-complete. By Proposition 2.14, the subspace V_x of X is densely fan-complete for every $x \in X$. By an argument similar to the proof of Proposition 2.14, for every $x \in X$ there exists a dense A-sieve $\mathcal{V}_x = \{\mathcal{V}_n(x) = \{V_\alpha(x) : \alpha \in A_n(x)\}, \pi_{n,x} : A_{n+1}(x) \to A_n(x) : n \in \omega\}$ on V_x with the Property (C1) and for every $n \in \omega$ and every $\alpha \in A_n(x)$, the set $\overline{V_\beta(x)} \subset V_\alpha(x)$ if $\beta \in \pi_{n,x}^{-1}(\alpha)$.

For every $n \in \omega$, we let $A_n = \bigcup\{A_n(x) \times \{x\} : x \in X\}$. For every $n \in \omega$, let $\pi_n : A_{n+1} \to A_n$ be a mapping such that if $\langle \alpha, x \rangle \in A_{n+1}(x) \times \{x\}$ for some $x \in X$, then $\pi_n(\langle \alpha, x \rangle) = \alpha$. For every $n \in \omega$ and every $x \in X$, let $U_{\langle \alpha, x \rangle} = V_\alpha(x)$ for every $\langle \alpha, x \rangle \in A_n(x) \times \{x\}$. Then $\mathcal{U} = \{\mathcal{U}_n = \{U_{\langle \alpha, x \rangle} : \langle \alpha, x \rangle \in A_n(x) \times \{x\}, x \in X\}, \pi_n : A_{n+1} \to A_n : n \in \omega\}$ is a dense A-sieve on X with the Property (C1). Thus X is densely fan-complete. □

The following notions appears in [12]. Let (X, τ) be a topological space and let D be a dense subset of X. On X we consider the *$\mathcal{G}_S(D)$-game* played between two players α and β. Player β goes first and chooses a non-empty open subset $B_1 \subset X$. Player α must then respond by choosing a non-empty open subset $A_1 \subset B_1$. Following this, player β must select another non-empty open subset $B_2 \subset A_1 \subset B_1$ and in turn player α must again respond by selecting a non-empty open subset $A_2 \subset B_2 \subset A_1 \subset B_1$. Continuing this procedure indefinitely the players α and β produce a sequence $((A_n, B_n) : n \in \mathbb{N})$ of pairs of open sets called a *play* of the $\mathcal{G}_S(D)$-game. We shall declare that α wins a play $((A_n, B_n) : n \in \mathbb{N})$ of the $\mathcal{G}_S(D)$-game if; $\bigcap_{n\in\mathbb{N}} A_n$ is non-empty and each sequence $(a_n : n \in \mathbb{N})$ with $a_n \in A_n \cap D$ has a cluster-point in X. Otherwise the player β is said to have won this play. By a *strategy* t for the player β we mean a '*rule*' that specifies each move of the player β in every possible situation. More precisely, a strategy $t := (t_n : n \in \mathbb{N})$ for β is a sequence of τ-valued functions such that $t_{n+1}(A_1, ..., A_n) \subset A_n$ for each $n \in \mathbb{N}$. The domain of each function t_n is precisely the set of all finite sequences $(A_1, A_2, ..., A_{n-1})$ of length $n-1$ in τ with $A_j \subset t_j(A_1, ..., A_{j-1})$ for all $1 \le j \le n-1$. The sequence of length 0 will be denoted by $\emptyset$. Such a finite sequence $(A_1, A_2, ..., A_{n-1})$ or infinite sequence $(A_n : n \in \mathbb{N})$ is called a *t-sequence*. A strategy $t := (t_n : n \in \mathbb{N})$ for the player β is called a *winning strategy* if each t-sequence is won by β. We will call a topological space (X, τ) a *strongly Baire* or (*strongly β-unfavorable*) space if it is regular and there exists a dense subset D of X such that the player β does not have a winning strategy in the $\mathcal{G}_S(D)$-game played on X [12]. In [2], the authors provided a large class of topological spaces X for which the absence of winning strategy for player β is equivalent to the requirement that X is a Baire space.

Theorem 2.21. *If X is a densely q-complete regular space, then X is a strongly Baire space.*

Proof. Since X is a densely q-complete space, there exist a dense subspace D of X and a dense A-sieve $\mathcal{U} = \{\gamma_n = \{U_\alpha : \alpha \in \Lambda_n\}, \pi_n : \Lambda_{n+1} \to \Lambda_n : n \in \omega\}$ with the Property (C2). Let us prove that X is a strongly Baire space. Let $t := (t_n : n \in \mathbb{N})$ be the strategy for player β. Let us construct a t-sequence $(A_n : n \in \mathbb{N})$ that wins for α. Let $B_1 = t_1(\emptyset)$. Then B_1 is a non-empty open subset of X.

Since $\bigcup\{U_\alpha : \alpha \in \Lambda_0\}$ is dense in X, there exists $\alpha_0 \in \Lambda_0$ such that $B_1 \cap U_{\alpha_0} \neq \emptyset$. Since X is regular and $B_1 \cap U_{\alpha_0}$ is a non-empty open subset of X, there exists a non-empty open subset A_1 of X such that $A_1 \subset \overline{A_1} \subset B_1 \cap U_{\alpha_0}$. Let $B_2 = t_2(A_1)$ be a non-empty open subset of X such that $B_2 \subset A_1$. Since $\bigcup\{U_\beta : \beta \in \pi^{-1}(\alpha_0)\}$ is dense in U_{α_0}, there exists $\alpha_1 \in \Lambda_1 \cap \pi_0^{-1}(\alpha_0)$ such that $U_{\alpha_1} \cap B_2 \neq \emptyset$. Then there exists a non-empty open subset A_2 of X such that $\overline{A_2} \subset U_{\alpha_1} \cap B_2$ by the regularity of X. Then $\overline{A_2} \subset A_1$ and $t_2(A_1) = B_2$. Take a non-empty open subset $B_3 = t_3(A_1, A_2) \subset A_2$.

Let $n \ge 1$. Assume that we have finite sequences $(B_1, ..., B_{n+1})$, $(A_1, ..., A_n)$, $(\alpha_0, ..., \alpha_{n-1})$ with the following properties:

1. $\overline{A_1} \subset B_1 \cap U_{\alpha_0}$;
2. $(B_1, ..., B_{n+1})$ and $(A_1, ..., A_n)$ are finite sequences of open subsets of X;
3. For each $0 \le i \le n-1$, $\alpha_i \in \Lambda_i$ and $\pi_i(\alpha_{i+1}) = \alpha_i$ for each $i \le n-2$;
4. For each $1 \le i \le n$, $\overline{A_i} \subset B_i \cap U_{\alpha_{i-1}}$;
5. $B_{i+1} = t_{i+1}(A_1, ..., A_i) \subset A_i$ for each $i \le n$.

Then $B_{n+1} = t_{n+1}(A_1, ..., A_n) \subset A_n \subset \overline{A_n} \subset B_n \cap U_{\alpha_{n-1}}$. Since $\bigcup\{U_\beta : \beta \in \pi_{n-1}^{-1}(\alpha_{n-1})\}$ is dense in $U_{\alpha_{n-1}}$, there exists $\alpha_n \in \Lambda_n \cap \pi_{n-1}^{-1}(\alpha_{n-1})$ such that $B_{n+1} \cap U_{\alpha_n} \neq \emptyset$. Then there exists a non-empty open subset A_{n+1} of X such that $\overline{A_{n+1}} \subset B_{n+1} \cap U_{\alpha_n}$. Let $B_{n+2} = t_{n+2}(A_1, ..., A_{n+1})$ be a non-empty open subset of X such that $B_{n+2} \subset A_{n+1}$.

By induction we get two sequences $\{B_n : n \in \mathbb{N}\}$ and $\{A_n : n \in \mathbb{N}\}$ of open subsets of X and a c-sequence $\alpha = \{\alpha_n : n \in \omega\}$ with the following properties:

1. $\overline{A_1} \subset B_1 \cap U_{\alpha_0}$;
2. $B_{n+1} = t_{n+1}(A_1, ..., A_n) \subset A_n$ for every $n \in \mathbb{N}$;
3. $\overline{A_{n+1}} \subset B_{n+1} \cap U_{\alpha_n}$ for every $n \in \mathbb{N}$.

Thus $\overline{A_{n+1}} \subset A_n \cap U_{\alpha_n}$ for every $n \in \mathbb{N}$. If $\{x_n\}_{n\in\omega}$ is a sequence of points of X such that $x_n \in A_{n+1} \cap D$ for every $n \in \omega$, then $x_n \in U_{\alpha_n} \cap D$ for every $n \in \omega$.

Since the dense subspace D and the dense A-sieve $\mathcal{U} = \{\gamma_n = \{U_\alpha : \alpha \in \Lambda_n\}, \pi_n : \Lambda_{n+1} \to \Lambda_n : n \in \omega\}$ satisfy the Property (C2), the sequence $\{x_n\}_{n\in\omega}$ has an accumulation point y in X and $y \in \bigcap\{\overline{A_n} : n \in \omega\}$. Then $\bigcap\{A_n : n \in \mathbb{N}\} = \bigcap\{\overline{A_n} : n \in \omega\} \neq \emptyset$. Thus the strategy $t := (t_n : n \in \mathbb{N})$ for the player β does not win. Then X is a strongly Baire space. □

3. Continuity in semitopological groups

All topological groups in this note are assumed to be Hausdorff. Let e be the identity of the considered group in this note. Given a semitopological group G, the symbol $\mathcal{N}(e)$ denotes the family of open neighborhoods of the identity e in G.

Lemma 3.1. ([12], Theorem 2) *Let G be a semitopological regular group. If G is a strongly Baire space, then G is a topological group.*

Theorem 3.2. *If G is a regular countably sieve-complete semitopological group, then G is a topological group.*

Proof. It can be gotten by Proposition 2.6, Theorem 2.21 and Lemma 3.1. It can also be gotten by Proposition 2.6 in this note and Theorem 5.2 in [1]. □

Corollary 3.3. *Let G be a regular semitopological group. If G is locally countably sieve-complete, then G is a topological group.*

Proof. By Lemma 2.17, G is countably sieve-complete. Then by Theorem 3.2 G is a topological group. □

By Theorem 3.2 and Proposition 2.19, we have the following result.

Corollary 3.4. *If G is a locally countably compact regular semitopological group, then G is a topological group.*

Theorem 3.5. *If G is a regular semitopological group with a densely q-complete dense subgroup, then G is a topological group.*

Proof. Let H be a densely q-complete dense subgroup of G. By Theorem 2.7, G is densely q-complete. Since every densely q-complete regular semitopological group is a topological group ([1], Theorem 5.2), the semitopological group G is a topological group. □

By Proposition 2.6 and Theorem 3.5, we have the following result.

Corollary 3.6. *If G is a regular semitopological group with a countably sieve-complete dense subgroup, then G is a topological group.*

In what follows, we show that if G is a regular countably sieve-complete semitopological group, then G is a D-space if and only if G is paracompact.

The following result was pointed out in ([15], p. 730).

Lemma 3.7. ([15], p. 730) *The following properties of a strong sieve $(\{U_\alpha : \alpha \in \Lambda_n\}, \pi_n)$ on a space X are equivalent:*

- *(a) $(\{U_\alpha : \alpha \in \Lambda_n\}, \pi_n)$ is a countably complete sieve;*
- *(b) If (α_n) is a π-chain, if $U_{\alpha_n} \neq \emptyset$ for all n, and if $C = \bigcap_{n\in\mathbb{N}} U_{\alpha_n}$, then C is nonempty, closed, and countably compact, and every open $V \supset C$ contains some U_{α_n}.*

Recall that a topological group G is *feathered* if it contains a non-empty compact set K with countable character in G ([4], p. 235).

Lemma 3.8. *Let $\mathcal{P}$ be a topological property such that every countably compact space X with property $\mathcal{P}$ is compact, property $\mathcal{P}$ is hereditary with respect to closed sets. If G is a regular countably sieve-complete semitopological group with property $\mathcal{P}$, then G is a paracompact Čech-complete topological group.*

Proof. By Theorem 3.2, G is a topological group. By Lemmas 2.4 and 3.7, there exists a non-empty countably compact closed subset K of G with countable character in G. Then K is countably compact and has property $\mathcal{P}$. Thus K is compact. Then G is feathered. Every feathered topological group is paracompact ([4], Corollary 4.3.21). Then G is paracompact countably sieve-complete. It follows from Lemma 2.8 G is Čech-complete. □

Theorem 3.9. *If G is a regular countably sieve-complete semitopological group, then G is a D-space if and only if G is paracompact.*

Proof. Assume that G is a D-space. Since the D-property is hereditary with respect to closed subsets and every countably compact T_1 D-space is compact, it follows from Lemma 3.8 G is a paracompact.

Now we assume that G is a paracompact countably sieve-complete semitopological group. By Lemma 2.8 and Theorem 3.2, G is a Čech-complete topological group. By ([4], Theorem 4.3.20), there exists a compact subgroup H of G such that G/H is a complete metric space. Let $\pi : G \to G/H$ be the canonical quotient homomorphism. By ([4], Theorem 1.5.7), the mapping π is perfect. Since every metric space is a D-space and every perfect preimage of a D-space is a D-space [5], it follows that G is a D-space. □

Given a paratopological group G with a topology τ, one defines the *conjugate topology* τ^{-1} on G by $\tau^{-1} = \{U^{-1} : U \in \tau\}$. The upper bounded $\tau^* = \tau \vee \tau^{-1}$ is a topological group topology. We call $G^* = (G, \tau^*)$ the *group associated to* G [23]. A paratopological group is called *totally* $\mathcal{P}$ if the associated topological group G^* has property $\mathcal{P}$ [23]. Recall that a semitopological group G is *ω-narrow* if for any neighborhood U of the identity e in G, there exists a countable set $C \subset G$ such that $CU = UC = G$.

In [22], Sánchez gave an internal characterization of subgroups of products of metrizable semitopological groups. A family $\mathcal{U}$ of subsets of a semitopological group G is *discrete with respect to a family* $\gamma \subset \mathcal{N}(e)$ if for every $x \in G$ we can find $V \in \gamma$ such that xV intersects at most one element of $\mathcal{U}$. Also, we say that $\mathcal{U}$ is *σ-discrete with respect to a family* $\gamma \subset \mathcal{N}(e)$ if $\mathcal{U}$ can be decomposed as a countable union of families discrete with respect to γ. The family $\mathcal{U}$ of subsets of G is *dominated by a family* $\gamma \subset \mathcal{N}(e)$ if for every $U \in \mathcal{U}$ and $x \in U$ there exists $V \in \gamma$ such that $xV \subset U$ [22]. Let $\mathcal{U}$ be a cover of a space X. We say that a refinement $\mathcal{V}$ of $\mathcal{U}$ is *basic* if for every $U \in \mathcal{U}$ and $x \in U$ there exists $V \in \mathcal{V}$ such that $x \in V \subset U$ [22]. A semitopological group has *property* (∗) if for every $U \in \mathcal{N}(e)$, the family $\{Ux : x \in G\}$ has an open basic refinement which is dominated by a countable family γ and σ-discrete with respect to γ ([22], Definition 2.3). The *symmetry number* of a T_1 semitopological group G, denoted by $Sm(G)$, is the minimum cardinal number κ such that for every neighborhood U of e in G, there exists a family $\mathcal{V}$ of neighborhoods of e in G such that $\bigcap_{V \in \mathcal{V}} V^{-1} \subset U$ and $|\mathcal{V}| \leq \kappa$ [21]. If G is a regular countably sieve-complete semitopological group with $Sm(G) \leq \omega$ and satisfies property (∗), then G is a topological group ([17], Theorem 2.14). By Theorem 3.2, the conditions of $Sm(G) \leq \omega$ and property (∗) in Theorem 2.14 in [17] is not essential.

In ([17], Corollary 2.15), it is proved that if G is a regular totally ω-narrow countably sieve-complete paratopological group, then G is a topological group. By Theorem 3.2, the property of totally ω-narrowness of the paratopological group G in Corollary 2.15 in [17] is not essential.

Acknowledgement

The author would like to thank the referee for his (or her) valuable remarks and suggestions which greatly improved the paper.

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