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Lacunary statistical ward continuity in metric spaces

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Abstract. In this paper, we introduce the concept of lacunary statistically p-quasi-Cauchyness of a sequence in a metric space in the sense that a sequence (x_k) is lacunary statistically p-quasi-Cauchy if $\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r : d(x_{k+p},x_k)\geqslant\varepsilon\}|=0$ for each $\varepsilon>0$ where $I_r=(k_{r-1},k_r]$ and $k_0=1$, $h_r=k_r-k_{r-1}\to\infty$ as $r\to\infty$ and $\theta=(k_r)$ is an increasing sequence of positive integers. A function f is called lacunary statistically p-ward continuous on a subset A of X if it preserves lacunary statistically p-quasi-Cauchy sequences, i.e. the sequence $(f(x_n))$ is lacunary statistically p-quasi-Cauchy whenever $\mathbf{x}=(x_n)$ is a lacunary statistically p-quasi-Cauchy sequence of points in A. It turns out that a function f is uniformly continuous on a totally bounded subset A of X if there exists a positive integer p such that f preserves lacunary statistically p-quasi-Cauchy sequences of points in A.

1. Introduction

The concept of continuity and any concept involving continuity play a very important role not only in pure mathematics but also in other branches of sciences involving mathematics especially in computer sciences, information theory, biological science, economics, and dynamical systems.

Throughout this paper, $\mathbb{N}$, $\mathbb{R}$, and X will denote the set of positive integers, the set of real numbers, and a metric space with a metric d, respectively. p will always be a fixed element of $\mathbb{N}$. The boldface letters such as $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ will be used for sequences $\mathbf{x}=(x_n)$, $\mathbf{y}=(y_n)$, $\mathbf{z}=(z_n)$, ... of points in X. A function $f : X \longrightarrow X$ is continuous if and only if it preserves convergent sequences. Using the idea of continuity of a real function in this manner, many kinds of continuities were introduced and investigated, not all but some of them we recall in the following: ward continuity ([2]), p-ward continuity ([5]), δ^2-ward continuity ([1]), statistical ward continuity, ([10]), [4]), slowly oscillating continuity ([15]), lacunary statistical ward continuity ([6], [16]), lacunary statistical δ ward continuity ([7]), lacunary statistical δ^2 ward continuity ([17]), which enabled some authors to obtain interesting results.

In [11] Fridy and Orhan introduced the concept of lacunary statistically convergence in the sense that a sequence (α_k) of points in $\mathbb{R}$ is called lacunary statistically convergent, or S_θ-convergent, to an element L of $\mathbb{R}$ if $\lim_{r\to\infty}\frac{1}{h_r}|\{k\in I_r : |\alpha_k-L|\geqslant\varepsilon\}|=0$ for every positive real number ε where $I_r=(k_{r-1},k_r]$ and $k_0=0$, $h_r=k_r-k_{r-1}\to\infty$ as $r\to\infty$ and $\theta=(k_r)$ is an increasing sequence of positive integers (see also [12], [3], [17], and [16]).

In the sequel, we shall assume that $\lim inf_r\ q_r>1$ where $q_r=\frac{k_r}{k_{r-1}}$. The set of lacunary statistically convergent sequences of points in X is denoted by $S_\theta(X)$. A sequence (x_k) of points in X is called lacunary

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statistically quasi-Cauchy if $S_\theta - lim \Delta\alpha_k = 0$, where $\Delta\alpha_k = d(x_{k+1}, x_k)$ for each positive integer k. The set of lacunary statistically quasi-Cauchy sequences in X will be denoted by $\Delta S_\theta(X)$.

The purpose of this paper is to introduce lacunary statistically p-quasi-Cauchy sequences in metric spaces, and prove concerning theorems.

2. Variations on lacunary statistical ward compactness

The concept of a Cauchy sequence involves far more than that the distance between successive terms is tending to 0 and lacunary statistically tending to zero, and more generally speaking, than that the distance between p-successive terms is lacunary statistically tending to zero, by p-successive terms we mean α_{k+p} and α_k. Nevertheless, sequences which satisfy this weaker property are interesting in their own right.

Before giving our main definition we recall basic concepts. A sequence (x_n) is called quasi Cauchy if $\lim_{n\to\infty} \Delta x_n = 0$, where $\Delta x_n = d(x_{n+1}, x_n)$ for each $n \in \mathbb{N}$ ([2]). A sequence (x_k) of points in X is slowly oscillating if $\lim_{\lambda\to 1^+} \overline{\lim}_n \max_{n+1\leqslant k\leqslant[\lambda n]} d(x_k, x_n) = 0$, where $[\lambda n]$ denotes the integer part of λn ([9]). A sequence (x_k) is quasi-slowly oscillating if (Δx_k) is slowly oscillating. A sequence (x_n) is called lacunary statistically convergent to an element L of X if $\lim_{r\to\infty} \frac{1}{h_r}|\{k \in I_r : d(x_k, L) \geqslant \varepsilon\}| = 0$ for each $\varepsilon > 0$ ([3], and [11]). Now we introduce the concept of a lacunary statistically p-quasi-Cauchy sequence.

Definition 2.1. A sequence (x_k) of points in X is called lacunary statistically p-quasi-Cauchy if $S_\theta - lim_{k\to\infty}\Delta_p x_k = 0$, i.e. $\lim_{r\to\infty} \frac{1}{h_r}|\{k \in I_r : \Delta_p x_k \geqslant \varepsilon\}| = 0$ for each $\varepsilon > 0$, where $\Delta_p x_k = d(x_{k+p}, x_k)$ for every $k \in \mathbb{N}$.

We will denote the set of all lacunary statistically p-quasi-Cauchy sequences by Δ_p^θ. We note that a sequence is lacunary statistically quasi-Cauchy when $p = 1$, i.e. lacunary statistically 1-quasi-Cauchy sequences are lacunary statistical quasi-Cauchy sequences. It follows from the inclusion

$\{k \in I_r : d(x_{k+p}, x_k) \geqslant \varepsilon\}$

$\subseteq \{k \in I_r : d(x_{k+p}, x_{k+p-1}) \geqslant \frac{\varepsilon}{p}\} \cup \{k \in I_r : d(x_{k+p-1}, x_{k+p-2}) \geqslant \frac{\varepsilon}{p}\} \cup ...$

$\cup\{k \in I_r : d(x_{k+2}, x_{k+1}) \geqslant \frac{\varepsilon}{p}\} \cup \{k \in I_r : d(x_{k+1}, x_k) \geqslant \frac{\varepsilon}{p}\}$.

that any lacunary statistically quasi-Cauchy sequence is also lacunary statistically p-quasi-Cauchy, but the converse is not always true as it can be seen by considering the sequence (x_k) defined by $(x_k) = (x, y, x, y, ..., x, y, ...)$ is lacunary statistically 2-quasi Cauchy which is not lacunary statistically quasi Cauchy if x is different from y. More examples can be seen in [13, Section 1.4] for the real case. It is clear that any Cauchy sequence is in $\bigcap_{p=1}^{\infty} \Delta_p^\theta(X)$, so that each $\Delta_p^\theta(X)$ is a sequence space containing the space $C(X)$ of Cauchy sequences. It should also be noted that $C(X)$ is a proper subset of $\Delta_p^\theta(X)$ for each $p \in \mathbb{N}$.

Definition 2.2. A subset A of X is called lacunary statistically p-ward compact if any sequence of points in A has a lacunary statistically p-quasi-Cauchy subsequence.

We note that this definition of lacunary statistically p-ward compactness cannot be obtained by any summability matrix in the sense of [8].

Since any lacunary statistically quasi-Cauchy sequence is lacunary statistically p-quasi-Cauchy we see that any lacunary statistically ward compact subset of X is lacunary statistically p-ward compact for any $p \in \mathbb{N}$. A finite subset of X is lacunary statistically p-ward compact, the union of finite number of lacunary statistically p-ward compact subsets of X is lacunary statistically p-ward compact, and the intersection of any family of lacunary statistically p-ward compact subsets of X is lacunary statistically p-ward compact. Furthermore any subset of a lacunary statistically p-ward compact set of X is lacunary statistically p-ward compact and any totally bounded subset of X is lacunary statistically p-ward compact. These observations above suggest to us the following.

Theorem 2.3. *A subset A of X is totally bounded if and only if there exists a $p \in \mathbb{N}$ such that A is lacunary statistically p-ward compact.*

Proof. Although the proof that totally boundedness implies lacunary statistically p-ward compactness follows from the fact that any sequence of points in a totally bounded subset of X has a Cauchy subsequence, we give a proof not only for completeness but also for anticipating how to see potential equivalent conditions for total boundedness. Take any sequence (x_n) of points in E. Since E can be covered by a finite number of subsets of X of diameter less than 1, one of these sets, which we denote by A_1, must contain x_n for infinitely many values of n. We may choose a positive integer n_1 such that $x_{n_1} \in A_1$. Then A_1 is totally bounded and hence it can be covered by a finite number of subsets of A_1 of diameter less than $\frac{1}{2}$. One of these subsets of A_1, which we denote by A_2, contains x_n for infinitely many n. Choose a positive integer n_2 such that $n_2 > n_1$ and $x_{n_2} \in A_2$. Since $A_2 \subset A_1$, it follows that $x_{n_2} \in A_1$ as well. Continuing in this way, we obtain, for any positive integer k, a subset A_k of A_{k-1} with *diameter* less than $\frac{1}{k}$ and a term $x_{n_k} \in A_k$ of the sequence (x_n) , where $n_k > n_{k-1}$. Since all x_{n_k}, $x_{n_{k+1}}$, $x_{n_{k+2}}$,...,$x_{n_{k+j}}$,... lie in A_k and the diameter of A_k is less than $\frac{1}{k}$, it follows that (x_{n_k}) is a lacunary p-quasi-Cauchy subsequence of the sequence (x_n). To prove that lacunary statistically p-ward compactness implies totally boundedness, suppose that E is not totally bounded. Then there exists an $\varepsilon > 0$ such that there does not exist a finite ε-net. Take any $x_1 \in E$. By the assumption that E is not totally bounded, the open ball $B_E(x_1, \varepsilon)$ is not equal to E, i.e. $B_E(x_1, \varepsilon) \neq E$, so there exists an $x_2 \in E$ such that $d_E(x_1, x_2) \geqslant \varepsilon$, i.e. $x_2 \notin B_E(x_1, \varepsilon)$, and $x_2 \in E$. Then $B_E(x_1, \varepsilon) \cup B_E(x_2, \varepsilon) \neq E$ otherwise $B_E(x_1, \varepsilon) \cup B_E(x_2, \varepsilon)$ would be a finite ε-net in E. Let $x_3 \notin B_E(x_1, \varepsilon) \cup B_E(x_2, \varepsilon)$ i.e. $d_E(x_1, x_2) \geqslant \varepsilon$, $d_E(x_1, x_3) \geqslant \varepsilon$, and $d_E(x_2, x_3) \geqslant \varepsilon$. Continuing the process in this manner, one can obtain a sequence (x_n) of points in E such that

$$x_n \notin B_E(x_1, \varepsilon) \cup B_E(x_2, \varepsilon) \cup ... \cup B_E(x_{n-1}, \varepsilon),\ (n = 2, 3, ...)$$

$$i.e.\ d_E(x_i, x_n) \geqslant \varepsilon\ (i = 1, 2, ..., n-1)\ and\ (n = 1, 2, ...),\ n \neq i.$$

The sequence (x_n) constructed in this manner has no lacunary statistically p-quasi-Cauchy subsequence. Thus this contradiction completes the proof of the theorem. □

Corollary 2.4. *A subset of X is lacunary statistically p-ward compact if and only if it is lacunary statistically q-ward compact for any* $p, q \in \mathbb{N}$.

It follows from Theorem 2.3 that lacunary statistically p-ward compactness of a subset of A of X coincides with either of the following kinds of compactness: p-ward compactness ([5, Theorem 2.3]), statistical ward compactness ([4]), slowly oscillating compactness ([15]).

Corollary 2.5. *Let E be a subset of X. The following statements are equivalent:*
(a) E is totally bounded.
(b) E is p-ward compact.
(c) E is slowly oscillating compact.
(d) E is statistically p-ward compact.
(e) E is lacunary statistically p-ward compact.

3. Variations on lacunary statistical ward continuity

In this section, we investigate connections between uniformly continuous functions and lacunary statistically p-ward continuous functions. A function $f : X \longrightarrow X$ is continuous if and only if it preserves lacunary statistically convergent sequences. Using this idea, we introduce lacunary statistical p-ward continuity.

Definition 3.1. A function f is called lacunary statistically p-ward continuous on a subset A of X if it preserves lacunary statistically p-quasi-Cauchy sequences, i.e. the sequence $(f(x_n))$ is lacunary statistically p-quasi-Cauchy whenever (x_n) is a lacunary statistically p-quasi-Cauchy sequence of points in A.

We see that this definition of lacunary statistically p-ward continuity can not be obtained by any summability matrix A (see [8]).

We note that the composite of two lacunary statistically p-ward continuous functions is lacunary statistically p-ward continuous.

In connection with lacunary statistically p-quasi-Cauchy sequences, slowly oscillating sequences, and convergent sequences the problem arises to investigate the following types of continuity of a function on X.

(Δ_p^θ) $(x_n) \in \Delta_p^\theta \Rightarrow (f(x_n)) \in \Delta_p^\theta$

($\Delta_p^\theta c$) $(x_n) \in \Delta_p^\theta \Rightarrow (f(x_n)) \in c$

(Δ^s) $(x_n) \in \Delta^s \Rightarrow (f(x_n)) \in \Delta^s$

(c) $(x_n) \in c \Rightarrow (f(x_n)) \in c$

(d) $(x_n) \in c \Rightarrow (f(x_n)) \in \Delta_p^\theta$

We see that (Δ_p^θ) is lacunary statistically p-ward continuity of f, and (c) states the ordinary continuity of f. It is easy to see that ($\Delta_p^\theta c$) implies (Δ_p^θ), and (Δ_p^θ) does not imply ($\Delta_p^\theta c$); Δ_p^θ implies (d), and (d) does not imply (Δ_p^θ); ($\Delta_p^\theta c$) implies (c) and (c) does not imply ($\Delta_p^\theta c$); and (c) implies (d).

Theorem 3.2. *If f is lacunary statistically p-ward continuous on a subset E of X for some $p \in \mathbb{N}$, then it is lacunary statistically ward continuous on E.*

Proof. If $p = 1$, then it is obvious. So we would suppose that $p > 1$. Take any lacunary statistically p-ward continuous function f on E. Let (x_k) be any lacunary statistical quasi-Cauchy sequence of points in E. Write

$$(\xi_i) = (x_1, x_1, ..., x_1, x_2, x_2, ..., x_2, ..., x_n, x_n, ..., x_n, ...),$$

where the same term repeats p times. The sequence

$$(x_1, x_1, ..., x_1, x_2, x_2, ..., x_2, ..., x_n, x_n, ..., x_n, ...)$$

is also lacunary statistically quasi-Cauchy so it is lacunary statistically p-quasi-Cauchy. By the lacunary statistically p-ward continuity of f, the sequence

$$(f(x_1), f(x_1), ..., f(x_1), f(x_2), f(x_2), ..., f(x_2), ..., f(x_n), f(x_n), ..., f(x_n), ...)$$

is lacunary statistically p-quasi-Cauchy, where the same term repeats p-times. Thus the sequence

$$(f(x_1), f(x_1), ..., f(x_1), f(x_2), f(x_2), ..., f(x_2), ..., f(x_n), f(x_n), ..., f(x_n), ...)$$

is also lacunary statistically p quasi-Cauchy. It is easy to see that $S_\theta - lim d(f(x_{n+1}), f(x_n))) = 0$, which completes the proof of the theorem. □

Corollary 3.3. *f is lacunary statistically p-ward continuous on a subset E of X, then it is continuous on E in the ordinary sense.*

Proof. The proof follows immediately, so is omitted. □

Theorem 3.4. *Lacunary statistical p-ward continuous image of any lacunary statistically p-ward compact subset of X is lacunary statistically p-ward compact.*

Proof. Let f be a lacunary statistically p-ward continuous function, and A be a lacunary statistically p-ward compact subset of X. Take any sequence $\boldsymbol{\beta} = (\beta_n)$ of terms in $f(E)$. Write $\beta_n = f(x_n)$ where $x_n \in E$ for each $n \in \mathbb{N}$, $\boldsymbol{\alpha} = (x_n)$. Lacunary statistically p-ward compactness of A implies that there is a lacunary statistically p-quasi-Cauchy subsequence $\boldsymbol{\xi} = (\xi_k) = (x_{n_k})$ of $\boldsymbol{\alpha}$. Since f is lacunary statistically p-ward continuous, $(t_k) = f(\boldsymbol{\xi}) = (f(\xi_k))$ is lacunary statistically p-quasi-Cauchy. Thus (t_k) is a lacunary statistically p-quasi-Cauchy subsequence of the sequence $f(\boldsymbol{\alpha})$. This completes the proof of the theorem. □

Theorem 3.5. *If f is uniformly continuous on a subset E of X, then $(f(x_n))$ is lacunary statistically p-quasi-Cauchy whenever (x_n) is a p-quasi-Cauchy sequence of points in E.*

Proof. Let (x_n) be any p-quasi-Cauchy sequence of points in E. Take any $\varepsilon > 0$. Uniform continuity of f on E implies that there exists a $\delta > 0$, depending on ε, such that $d(f(x), f(y)) < \varepsilon$ whenever $d(x, y) < \delta$ and $x, y \in E$. For this $\delta > 0$, there exists an $N = N(\delta)$ such that $\Delta_p x_n < \delta$ whenever $n > N$. Hence $\Delta_p f(x_n) < \varepsilon$ if $n > N$. Thus $\{k \in I_r : \Delta_p f(x_k)| \geqslant \varepsilon\} \subseteq \{1, 2, ..., N\}$. Therefore

$\lim_{r\to\infty} \frac{1}{h_r}|\{k \in I_r : \Delta_p f(x_k)| \geqslant \varepsilon\}| \leqslant \lim_{r\to\infty, k(r)\geqslant N} \frac{1}{h_r}|\{k \in I_r : \Delta_p f(x_k)| = 0.$

It follows from this that $(f(x_n))$ is a lacunary statistically p-quasi-Cauchy sequence. This completes the proof of the theorem. □

It is well-known that any continuous function on a compact subset E of X is uniformly continuous on E. We have an analogous theorem for a lacunary statistically p-ward continuous function defined on a lacunary statistically p-ward compact subset of X.

Theorem 3.6. *If a function is lacunary statistically p-ward continuous on a lacunary statistically p-ward compact subset of X, then it is uniformly continuous on E.*

Proof. Suppose that f is not uniformly continuous on E so that there exist an $\epsilon_0 > 0$ and sequences (x_n) and (β_n) of points in E such that $d(x_n, \beta_n) < 1/n$ and $d(f(x_n); f(\beta_n)) \geqslant \epsilon_0$ for all $n \in \mathbb{N}$. Since E is lacunary statistically p-ward compact, there is a subsequence (x_{n_k}) of (x_n) that is lacunary statistically p-quasi-Cauchy. On the other hand, there is a subsequence $(\beta_{n_{k_j}})$ of (β_{n_k}) that is lacunary statistically p-quasi-Cauchy as well. It is clear that the corresponding sequence $(a_{n_{k_j}})$ is also lacunary statistically p-quasi-Cauchy, since

$\{j \in I_r : d(x_{n_{k_{j+p}}}, x_{n_{k_j}}) \geqslant \varepsilon\} \subseteq \{j \in I_r : d(x_{n_{k_{j+p}}}, \beta_{n_{k_{j+p}}}) \geqslant \frac{\varepsilon}{3}\} \cup \{j \in I_r : d(\beta_{n_{k_{j+p}}}, \beta_{n_{k_j}}) \geqslant \frac{\varepsilon}{3}\} \cup \{j \in I_r : d(\beta_{n_{k_j}}, x_{n_{k_j}}) \geqslant \frac{\varepsilon}{3}\}$

for every $n \in \mathbb{N}$, and for every $\varepsilon > 0$. Hence it is easy to establish a contradiction. Thus this completes the proof of the theorem. □

Corollary 3.7. *If a function defined on a totally bounded subset of X is lacunary statistically p-ward continuous, then it is uniformly continuous.*

We note that when the domain of a function is restricted to a totally bounded subset of X, lacunary statistically p-ward continuity implies not only ward continuity, but also statistically p-ward continuity.

4. Conclusion

In this paper, we introduce lacunary statistically p-quasi Cauchy sequences, and investigate conditions for a lacunary statistically p ward continuous function on a metric space to be uniformly continuous, and prove some other results related to these kinds of continuities and some other kinds of continuities. It turns out that lacunary statistically p-ward continuity implies uniform continuity on a totally bounded subset of X. The results in this paper not only involves the related results in [6] as a special case for $p = 1$, but also some interesting results which are also new for the special case $p = 1$. The lacunary statistically p-quasi Cauchy concept for $p > 1$ might find more interesting applications than lacunary statistical quasi Cauchy sequences to the cases when lacunary statistically quasi Cauchy does not apply. For a further study, we suggest to investigate lacunary statistically p-quasi-Cauchy sequences of fuzzy points, or soft points. However due to the change in settings, the definitions and methods of proofs will not always be analogous to those of the present work. We also suggest to investigate lacunary statistically p-quasi-Cauchy double sequences of points in X (see [14] for the related definitions in the double case). For another further study, we suggest to investigate lacunary statistically p-quasi-Cauchy sequences in abstract metric spaces.

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