



## Common fixed point of interpolative Hardy-Rogers pair contraction

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**Abstract.** This paper aimed to obtain common fixed point results by using an interpolative contraction condition given by Karapinar in the setting of complete metric space. Here in this paper, we introduce the notion of interpolative Hardy-Rogers pair-type contraction and prove the corresponding common fixed point theorem by adopting the notion of interpolation. The results are further validated with the application based on them.

### 1. Introduction

The Banach contraction principle effectively encapsulates and reinterprets the successive approximation techniques that were initially pioneered by several earlier mathematicians, including notable names like Cauchy, Liouville, Picard, Lipschitz, and others. Subsequently, the fundamental proposition presented in [13] has undergone modifications and has been explored in various directions.

In certain generalizations of the contraction mapping principle, the original inequality is relaxed, as evident in [15]. Conversely, some variations weaken the topological structure of the underlying space, as exemplified by [16] and its accompanying references. Amidst these explorations, a notable refinement of the Banach fixed point theorem was introduced by Hardy-Rogers [14]. The foundational embodiment of this advancement, as outlined in [14], is expressed as follows.

**Theorem 1.1.** [14] Let  $(E, d)$  be a complete metric space and  $T$  be a self-mapping of  $E$  satisfying the condition for all  $x, y \in E$ ,  $d(Tx, Ty) \leq ad(x, Tx) + bd(y, Ty) + cd(x, Tx) + ed(y, Ty) + fd(x, y)$

Where  $a, b, c, e, f$  are non-negative and  $a + b + c + e + f < 1$ . Then has a unique fixed point in  $E$ .

On the interpolative Hardy-Rogers type contractive mapping" and its generalization of Hardy-Rogers' fixed point theorem. It's interesting to note that E. Karapinar introduced this new type of mapping by incorporating the concept of interpolation into the Hardy-Rogers framework. This approach likely allows for the generation of intermediate points between known data points and expands the applicability of the original theorem.

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2020 Mathematics Subject Classification. Primary 47H10; Secondary 47H09, 54H25.

Keywords. Common fixed point, Interpolative Hardy-Rogers, metric space.

Received: 31 August 2023; Revised: 31 December 2023; Accepted: 30 January 2024

Communicated by Erdal Karapinar

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The use of interpolation to generalize various forms of contractions is indeed a common practice in mathematical research. By integrating interpolation techniques into contraction mappings, researchers can extend the scope of existing theorems and provide a more flexible framework for analyzing fixed points in metric spaces.

It seems that the interpolative method has been employed in other research as well as to generalize different types of contractions. This demonstrates the versatility and effectiveness of the interpolation approach in expanding the theory of fixed points and providing new insights into the existence and uniqueness of solutions.

To delve further into the specific details and implications of Karapinar's work and the generalization of other forms of contractions using the interpolative method, I recommend referring to the cited paper [1–8] and exploring related research in the field. These sources should provide a more comprehensive understanding of the interpolative Hardy-Rogers type contractive mapping and its applications in fixed point theory.

In 2018 Karapinar [3] proposed a new Kannan-type contractive mapping using the concept of interpolation and proved a fixed point theorem in metric space. This new type of mapping, called interpolative Kannan-type contractive mapping is a generalization of Kannan's fixed point theorem.

**Theorem 1.2.** *Let us recall that an interpolative Kannan contraction on a metric space  $(E, d)$  is a self-mapping  $T : E \rightarrow E$  such that there exist  $k \in [0, 1)$  and  $\alpha \in (0, 1)$  such that*

$$d(Tx, Ty) \leq k [d(Tx, x)]^\alpha [d(Ty, y)]^{1-\alpha}, \quad (1)$$

$(x, y) \in E \times E$  with  $x, y \notin \text{Fix}(T)$

Then  $T$  has a unique fixed point in  $E$ .

Following this, Karapinar et al [1] proposed a new Hardy-Rogers type contractive mapping using the concept of interpolation and proving a fixed point theorem in metric space. This new type of mapping, called "interpolative Hardy-Rogers type contractive mapping" is a generalization of Hardy-Rogers's fixed point theorem.

**Definition 1.3.** [1] *Let  $(E, d)$  be a metric space. We say that the self-mapping  $T : X \rightarrow X$  is said to be a interpolative Hardy-Rogers type contraction if there exists  $k \in [0, 1)$  and  $\alpha, \beta, \gamma \in (0, 1)$  with  $\alpha + \beta + \gamma < 1$ , such that*

$$d(Tx, Ty) \leq k [d(x, y)]^\beta [d(Tx, x)]^\alpha [d(Ty, y)]^\gamma \cdot \left[ \frac{1}{2} (d(Tx, y) + d(Ty, x)) \right]^{1-\alpha-\beta-\gamma} \quad (2)$$

for all  $x, y \in E \setminus \text{Fix}(T)$ .

**Theorem 1.4.** [1] *Let  $(E, d)$  be a complete metric space and  $T$  be an interpolative Hardy-Rogers type contraction. Then,  $T$  has a fixed point in  $E$ .*

In this paper, we introduce the concept of interpolative Hardy-Rogers pair contractions and demonstrate their effectiveness through illustrative examples.

## 2. Main results

In this section, we are following interpolative Hardy-Rogers result in [1] to obtain a common fixed point result.

**Definition 2.1.** Let  $(E, d)$  be a metric space. A pair of mappings  $T, S : X \rightarrow X$  is said to be interpolative Hardy-Rogers pair contraction if there exists  $k \in [0, 1)$  and  $\alpha, \beta, \gamma \in (0, 1)$  with  $\alpha + \beta + \gamma < 1$ , such that

$$d(Tx, Sy) \leq k [d(x, y)]^\beta [d(Tx, x)]^\gamma [d(Sy, y)]^\alpha \cdot \left[ \frac{1}{2} (d(Tx, y) + d(Sy, x)) \right]^{1-\alpha-\beta-\gamma} \tag{3}$$

for all  $x, y \in E$  such that  $Tx \neq x$  whenever  $Sy \neq y$ .

**Theorem 2.2.** Suppose that  $(E, d)$  be a complete metric space, and  $(T, S)$  is interpolative Hardy-Rogers pair. Then,  $S$  and  $T$  have a unique common fixed point.

*Proof.* Starting from  $x_0 \in X$ , consider  $\{x_n\}$ , given as  $x_{2n+1} = Tx_{2n}$  and  $x_{2n+2} = Sx_{2n+1}$  for each positive integer  $n$

Take  $x = x_{2n}$  and  $y = x_{2n+1}$  in (3), we get

$$\begin{aligned} d(x_{2n+1}, x_{2n+2}) &= d(Tx_{2n}, Sx_{2n+1}) \leq k [d(x_{2n}, x_{2n+1})]^\beta [d(x_{2n}, Tx_{2n})]^\gamma [d(x_{2n+1}, Sx_{2n+1})]^\alpha \\ &\quad \cdot \left[ \frac{1}{2} (d(Tx_{2n}, x_{2n+1}) + d(Sx_{2n+1}, x_{2n})) \right]^{1-\alpha-\beta-\gamma} \\ &\leq k [d(x_{2n}, x_{2n+1})]^{\beta+\gamma} [d(x_{2n+1}, x_{2n+2})]^\alpha \cdot \\ &\quad \left[ \frac{1}{2} (d(x_{2n+1}, x_{2n+1}) + d(x_{2n+2}, x_{2n})) \right]^{1-\alpha-\beta-\gamma} \end{aligned}$$

Then

$$\begin{aligned} d(x_{2n+1}, x_{2n+2}) &\leq k [d(x_{2n}, x_{2n+1})]^{\beta+\gamma} [d(x_{2n+1}, x_{2n+2})]^\alpha \cdot \left[ \frac{1}{2} d(x_{2n+2}, x_{2n}) \right]^{1-\alpha-\beta-\gamma} \\ [d(x_{2n+1}, x_{2n+2})]^{1-\alpha} &\leq k [d(x_{2n}, x_{2n+1})]^{\beta+\gamma} \cdot \left[ \frac{1}{2} (d(x_{2n+2}, x_{2n+1}) + d(x_{2n+1}, x_{2n})) \right]^{1-\alpha-\beta-\gamma} \end{aligned} \tag{4}$$

Suppose that  $d(x_{2n+1}, x_{2n}) < d(x_{2n+2}, x_{2n+1})$ .

Thus,

$$\begin{aligned} d(x_{2n+1}, x_{2n}) + d(x_{2n+2}, x_{2n+1}) &< d(x_{2n+2}, x_{2n+1}) + d(x_{2n+2}, x_{2n+1}) \\ d(x_{2n+1}, x_{2n}) + d(x_{2n+2}, x_{2n+1}) &< 2d(x_{2n+2}, x_{2n+1}) \\ \frac{1}{2} [d(x_{2n+1}, x_{2n}) + d(x_{2n+2}, x_{2n+1})] &< d(x_{2n+2}, x_{2n+1}) \end{aligned}$$

Consequently, the inequality (4) yields that

$$\begin{aligned} [d(x_{2n+1}, x_{2n+2})]^{1-\alpha} &\leq k [d(x_{2n}, x_{2n+1})]^{\beta+\gamma} \cdot [d(x_{2n+2}, x_{2n+1})]^{1-\alpha-\beta-\gamma} \\ [d(x_{2n+1}, x_{2n+2})]^{\beta+\gamma} &\leq k [d(x_{2n}, x_{2n+1})]^{\beta+\gamma} \end{aligned}$$

which implies that

$$d(x_{2n+1}, x_{2n+2}) < kd(x_{2n}, x_{2n+1})$$

So, we conclude that  $d(x_{2n+1}, x_{2n}) > d(x_{2n+2}, x_{2n+1})$ , which is a contradiction. Thus, we have  $d(x_{2n+1}, x_{2n+2}) < kd(x_{2n}, x_{2n+1})$

Then  $\frac{1}{2} (d(x_{2n+2}, x_{2n+1}) + d(x_{2n+1}, x_{2n})) \leq d(x_{2n+1}, x_{2n})$ . Consequently, the inequality (4) yields that

$$\begin{aligned} [d(x_{2n+1}, x_{2n+2})]^{1-\alpha} &\leq k [d(x_{2n}, x_{2n+1})]^{\beta+\gamma} \cdot [d(x_{2n+1}, x_{2n})]^{1-\alpha-\beta-\gamma} \\ [d(x_{2n+1}, x_{2n+2})]^{1-\alpha} &\leq k [d(x_{2n+1}, x_{2n})]^{1-\alpha} \\ &\leq kd(x_{2n+1}, x_{2n}) \\ &\vdots \\ &\leq k^{2n+1}d(x_0, x_1) \end{aligned}$$

We deduce that,

$$d(x_{2n+1}, x_{2n+2}) \leq k^{2n+1}d(x_0, x_1) \tag{5}$$

Similarly, for  $x = x_{2n}$  and  $y = x_{2n-1}$  in (3), we obtain that

$$\begin{aligned} d(x_{2n+1}, x_{2n}) &= d(Tx_{2n}, Sx_{2n-1}) \\ &\leq k [d(x_{2n}, x_{2n-1})]^\beta [d(x_{2n}, Tx_{2n})]^\gamma [d(x_{2n-1}, Sx_{2n-1})]^\alpha \cdot \\ &\quad \left[ \frac{1}{2} (d(Tx_{2n}, x_{2n-1}) + d(Sx_{2n-1}, x_{2n})) \right]^{1-\alpha-\beta-\gamma} \\ &\leq k [d(x_{2n}, x_{2n-1})]^\beta [d(x_{2n}, x_{2n+1})]^\gamma [d(x_{2n-1}, x_{2n})]^\alpha \cdot \\ &\quad \left[ \frac{1}{2} (d(x_{2n+1}, x_{2n-1}) + d(x_{2n}, x_{2n})) \right]^{1-\alpha-\beta-\gamma} \end{aligned}$$

Then

$$d(x_{2n+1}, x_{2n}) \leq k [d(x_{2n}, x_{2n-1})]^{\alpha+\beta} [d(x_{2n}, x_{2n+1})]^\gamma \cdot \left[ \frac{1}{2} (d(x_{2n+1}, x_{2n-1})) \right]^{1-\alpha-\beta-\gamma}$$

We deduce that

$$[d(x_{2n+1}, x_{2n})]^{1-\gamma} \leq k [d(x_{2n}, x_{2n-1})]^{\alpha+\beta} \cdot \left[ \frac{1}{2} (d(x_{2n+1}, x_{2n}) + d(x_{2n}, x_{2n-1})) \right]^{1-\alpha-\beta-\gamma} \tag{6}$$

Suppose that  $d(x_{2n}, x_{2n-1}) < d(x_{2n+1}, x_{2n})$ .

Thus,

$$\begin{aligned} d(x_{2n}, x_{2n-1}) + d(x_{2n+1}, x_{2n}) &< d(x_{2n+1}, x_{2n}) + d(x_{2n+1}, x_{2n}) \\ d(x_{2n}, x_{2n-1}) + d(x_{2n+1}, x_{2n}) &< 2d(x_{2n+1}, x_{2n}) \\ \frac{1}{2}(d(x_{2n}, x_{2n-1}) + d(x_{2n+1}, x_{2n})) &< d(x_{2n+1}, x_{2n}) \end{aligned}$$

Consequently, the inequality (6) yields that

$$\begin{aligned} [d(x_{2n+1}, x_{2n})]^{1-\gamma} &\leq k [d(x_{2n}, x_{2n-1})]^{\alpha+\beta} \cdot [d(x_{2n+1}, x_{2n})]^{1-\alpha-\beta-\gamma} \\ [d(x_{2n+1}, x_{2n})]^{\alpha+\beta} &\leq k [d(x_{2n}, x_{2n-1})]^{\alpha+\beta} \end{aligned}$$

which implies that

$$[d(x_{2n+1}, x_{2n})]^{\alpha+\beta} \leq [d(x_{2n}, x_{2n-1})]^{\alpha+\beta}$$

So, we conclude that  $d(x_{2n+1}, x_{2n}) < d(x_{2n}, x_{2n-1})$ , which is a contradiction. Thus, we have  $d(x_{2n+1}, x_{2n}) < d(x_{2n}, x_{2n-1})$

Then  $\frac{1}{2}(d(x_{2n}, x_{2n-1}) + d(x_{2n+1}, x_{2n})) \leq d(x_{2n}, x_{2n-1})$ . Consequently, the inequality (6) yields that

$$\begin{aligned} [d(x_{2n+1}, x_{2n})]^{1-\gamma} &\leq k [d(x_{2n}, x_{2n-1})]^{\alpha+\beta} \cdot [d(x_{2n}, x_{2n-1})]^{1-\alpha-\beta-\gamma} \\ &\leq [kd(x_{2n}, x_{2n-1})]^{1-\gamma} \\ &\leq kd(x_{2n}, x_{2n-1}) \\ &\vdots \\ &\leq k^{2n}d(x_0, x_1) \end{aligned}$$

We deduce that

$$d(x_{2n+1}, x_{2n}) \leq k^{2n}d(x_0, x_1) \tag{7}$$

It follows from (5) and (7), we deduce that

$$d(x_n, x_{n+1}) \leq k^n d(x_0, x_1) \tag{8}$$

For  $m > 0$ . Using the triangular inequality, we obtain

$$\begin{aligned} d(x_n, x_{n+m}) &\leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \dots + d(x_{n+m-1}, x_{n+m}) \\ &\leq k^n d(x_0, x_1) + k^{n+1}d(x_0, x_1) + \dots + k^{n+m-1}d(x_0, x_1) \\ &\leq (k^n + k^{n+1} + \dots + k^{n+m-1})d(x_0, x_1) \\ &\leq \frac{k^n}{1-k}d(x_0, x_1) \end{aligned}$$

Taking  $n \rightarrow +\infty$  in the inequality above, we derive that  $\{x_n\}$  is a Cauchy sequence. by completeness of  $(E, d)$ , here exists  $x^* \in X$  such that  $\lim_{n \rightarrow \infty} x_n = x^*$ . Using the continuity of the metric in its both variables, we can prove that  $x^*$  is a fixed point of  $T$  as follows

$$\begin{aligned} d(Tx^*, x_{2n+2}) &= d(Tx^*, Sx_{2n+1}) \\ &\leq k [d(x^*, x_{2n+1})]^\beta [d(Tx^*, x^*)]^\gamma [d(Sx_{2n+1}, x_{2n+1})]^\alpha \\ &\quad \left[ \frac{1}{2} (d(Tx^*, x_{2n+1}) + d(Sx_{2n+1}, x^*)) \right]^{1-\alpha-\beta-\gamma} \\ &\leq k [d(x^*, x_{2n+1})]^\beta [d(Tx^*, x^*)]^\gamma [d(x_{2n+2}, x_{2n+1})]^\alpha \\ &\quad \left[ \frac{1}{2} (d(Tx^*, x_{2n+1}) + d(x_{2n+2}, x^*)) \right]^{1-\alpha-\beta-\gamma} \end{aligned}$$

Letting  $n \rightarrow +\infty$  we get  $d(Tx^*, x^*) = 0$  that is  $Tx^* = x^*$ .

Similarly,

$$\begin{aligned} d(x_{2n+1}, Sx^*) &= d(Tx_{2n}, Sx^*) \\ &\leq k [d(x_{2n}, x^*)]^\beta [d(Tx_{2n}, x_{2n})]^\gamma [d(Sx^*, x^*)]^\alpha \\ &\quad \left[ \frac{1}{2} (d(Tx_{2n}, x^*) + d(Sx^*, x_{2n})) \right]^{1-\alpha-\beta-\gamma} \\ &\leq k [d(x_{2n}, x^*)]^\beta [d(x_{2n+1}, x_{2n})]^\gamma [d(Sx^*, x^*)]^\alpha \\ &\quad \left[ \frac{1}{2} (d(x_{2n+1}, x^*) + d(Sx^*, x_{2n})) \right]^{1-\alpha-\beta-\gamma} \end{aligned}$$

Letting  $n \rightarrow +\infty$  we get  $d(x^*, Sx^*) = 0$  that is  $Sx^* = x^*$ .

To establish the uniqueness of  $x^*$  as the shared fixed point of  $S$  and  $T$ , assume the existence of another common fixed point  $\bar{x}$  for  $S$  and  $T$ . Then,

$$d(x^*, \bar{x}) = d(Tx^*, S\bar{x}) \leq k [d(x^*, \bar{x})]^\beta [d(Tx^*, x^*)]^\gamma [d(S\bar{x}, \bar{x})]^\alpha \cdot \left[ \frac{1}{2} (d(Tx^*, \bar{x}) + d(S\bar{x}, x^*)) \right]^{1-\alpha-\beta-\gamma}$$

Hence  $d(x^*, \bar{x}) = 0$ , so  $x^* = \bar{x}$ .  $\square$

**Example 2.3.** Let  $E = \{a, b, x, y\}$  be endowed with the metric defined by the following

$$d(a, a) = d(b, b) = d(x, x) = d(y, y)$$

$$d(a, b) = d(b, a) = 3$$

$$d(a, x) = d(x, a) = 4$$

$$d(b, x) = d(x, b) = \frac{3}{2}$$

$$d(a, y) = d(y, a) = \frac{5}{2}$$

$$d(b, y) = d(y, b) = 2$$

$$d(x, y) = d(y, x) = \frac{3}{2}$$

Define self maps  $T, S$  as follows

$$T : \begin{pmatrix} a & b & x & y \\ a & y & x & y \end{pmatrix}, S : \begin{pmatrix} a & b & x & y \\ a & b & y & x \end{pmatrix}$$

Choose  $k = \frac{8}{9}$ ,  $\alpha = \frac{1}{3}$ ,  $\beta = \frac{1}{2}$  and  $\gamma = \frac{1}{7}$ . Then, we have to check that (3) holds.

Case-I

$$d(Tb, Sx) = d(y, y) = 0$$

$$d(b, x) = \frac{3}{2}$$

$$d(Tb, b) = d(y, b) = 2$$

$$d(Sx, x) = d(y, x) = \frac{3}{2}$$

$$d(Tb, x) = d(y, x) = \frac{3}{2}$$

$$d(Sx, b) = d(y, b) = 2$$

$$\frac{1}{2} [d(b, x)]^{\frac{1}{2}} [d(Tb, b)]^{\frac{1}{2}} [d(Sx, x)]^{\frac{1}{3}} \cdot \left[ \frac{1}{2} (d(Tb, x) + d(Sx, b)) \right]^{\frac{1}{42}} =$$

$$\frac{8}{9} \left[ \frac{3}{2} \right]^{\frac{1}{2}} [2]^{\frac{1}{2}} \left[ \frac{3}{2} \right]^{\frac{1}{3}} \cdot \left[ \frac{1}{2} \left( \frac{3}{2} + 2 \right) \right]^{\frac{1}{42}} \approx 1,394$$

Therefore, (3) holds.

Case-II

$$d(Tb, Sy) = d(y, x) = \frac{3}{2} = 1.5$$

$$d(b, y) = 2$$

$$d(Tb, b) = d(y, b) = 2$$

$$d(Sy, y) = d(x, y) = \frac{3}{2}$$

$$d(Tb, y) = d(y, y) = 0$$

$$d(Sy, b) = d(x, b) = \frac{3}{2}$$

$$\frac{1}{2} [d(b, y)]^{\frac{1}{2}} [d(Tb, b)]^{\frac{1}{2}} [d(Sy, y)]^{\frac{1}{3}} \cdot \left[ \frac{1}{2} (d(Tb, y) + d(Sy, b)) \right]^{\frac{1}{42}} =$$

$$\frac{8}{9} [2]^{\frac{1}{2}} [2]^{\frac{1}{2}} \left[ \frac{3}{2} \right]^{\frac{1}{3}} \cdot \left[ \frac{1}{2} \left( 0 + \frac{3}{2} \right) \right]^{\frac{1}{42}} \approx 1.577. \text{ Thus, (3) holds}$$

From all the above three cases, we obtain that  $(T, S)$  is interpolative Hardy-Rogers pair. Thus, by Theorem 2.2, Hence,  $a$  is the common fixed point of  $T$  and  $S$ .

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