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Some new extensions of Darbo's fixed point theorem with application

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Abstract. We present some new extensions of Darbo's fixed-point theorem with a different aspect than those available in the literature by introducing the concept of (k, L)-set contraction. The concept of (k, L)-set contraction is interesting since there is no restriction for the constant L. We also present the nonlinear Leray-Schauder type alternative for one of our results. Finally, to demonstrate the applicability, we utilize theoretical results to provide the existence of solution of an integral equation.

1. Introduction and Preliminaries

It is well known that the Schauder fixed point theorem, which is the counterpart of Brouwer's theorem in infinite dimensional spaces, especially in function spaces, has a very important place in nonlinear analysis. There are many generalizations of Schauder fixed point theorem in the literature [10, 11, 14, 22, 23]. One of the tools for dealing with nonlinear analysis is measure of the noncompactness. This technique is very useful in the existence of solution of differential and integral equations. In 1955 Darbo [13], considering the measure of noncompactness defined by Kuratowski [16], obtained an applicable fixed point theorem. Since then, some extensions of Darbo's fixed point theorem have been obtained. [1–5, 7–9, 12, 15, 17–20].

In this paper, we present some new extensions of Darbo's fixed point theorem with different aspects than in the previous papers. We also provide the nonlinear Leray-Schauder type alternative for one of our results. Finally, to demonstrate the applicability, we utilize theoretical results to provide the existence of solution of an integral equation.

First, we recall the fundamental definition and important properties of measure of noncompactness. (See for more information [6]).

Let $(\mathcal{X}, d)$ be a complete metric space and $\mathcal{B}(\mathcal{X})$ be the family of all bounded subsets of $\mathcal{X}$. A nonnegative real valued mapping μ defined on $\mathcal{B}(\mathcal{X})$ is called measure of noncompactness on $\mathcal{X}$ if it satisfies the following axioms: for all $E, E_1, E_2 \in \mathcal{B}(\mathcal{X})$,

- $\mu(E) = 0$ if and only if E is precompact set,
- $\mu(E) = \mu(\overline{E})$,
- $\mu(E_1 \cup E_2) = \max\{\mu(E_1), \mu(E_2)\}$.

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Let μ be a measure of noncompactness of a complete metric space $\mathcal{X}$, then the following properties hold: for all $E, E_1, E_2 \in \mathcal{B}(\mathcal{X})$,

- If $E_1 \subseteq E_2$, then $\mu(E_1) \leq \mu(E_2)$,
- $\mu(E_1 \cap E_2) \leq \min\{\mu(E_1), \mu(E_2)\}$,
- If E is a finite set, then $\mu(E) = 0$,
- (Generalized Cantor intersection property) If $\{E_n\}$ is a decreasing sequence of nonempty, closed and bounded subsets of $\mathcal{X}$ with $\mu(E_n) \to 0$ as $n \to \infty$, then $\bigcap_{n=1}^{\infty} E_n$ is nonempty and compact.

Further, if $\mathcal{X}$ is a Banach space, then the function μ has additional properties, some of which are given below: for all $E, E_1, E_2 \in \mathcal{B}(\mathcal{X})$,

- $\mu(\lambda E) = |\lambda| \mu(E)$, for any number λ,
- $\mu(E_1 + E_2) \leq \max\{\mu(E_1), \mu(E_2)\}$,
- $\mu(\xi_0 + E) = \mu(E)$ for any $\xi_0 \in \mathcal{X}$,
- $\mu(coE) = \mu(E)$, where coE is the convex hull of E.

Let us recall the fundamental fixed point theorem of Schauder.

Theorem 1.1 ([21] Schauder's fixed point theorem). *Let E be a nonempty, bounded, closed, and convex subset of a Banach space $\mathcal{X}$. Then each continuous and compact map $\mathcal{F} : E \to E$ has at least one fixed point in E.*

Darbo used the measure of noncompactness to introduce the following concept of k-set contraction to weaken the compactness condition in Schauder's theorem.

Definition 1.2. *Let E be a nonempty subset of a Banach space $\mathcal{X}$. A mapping $\mathcal{F} : E \to E$ is said to be a k-set contraction if, for each $D \subseteq E$ with bounded, $\mathcal{F}D$ is bounded and there exists $k \in [0, 1)$ such that*

$$\mu(\mathcal{F}D) \leq k\mu(D). \tag{1}$$

Since every compact mapping is a k-set contraction, the following theorem is more general than Schauder's.

Theorem 1.3 ([13] Darbo's fixed point theorem). *Let E be a nonempty, bounded, closed, and convex subset of a Banach space $\mathcal{X}$. Then each continuous k-set contraction $\mathcal{F} : E \to E$ has at least one fixed point in E.*

2. Main Results

We begin this section by introducing the concept of (k, L)-set contraction, which plays a crucial role in our main results.

Definition 2.1. *Let E be a nonempty subset of a Banach space $\mathcal{X}$. A mapping $\mathcal{F} : E \to E$ is said to be a (k, L)-set contraction if, for each $D \subseteq E$ with bounded, $\mathcal{F}D$ is bounded and there exist $k \in [0, 1)$ and $L \geq 0$ such that*

$$\mu(\mathcal{F}D) \leq k\mu(D) + L\left[\mu(D \cup \mathcal{F}D) - \mu(D \cap \mathcal{F}D)\right]. \tag{2}$$

It is clear that every k-set contraction is also a (k, L)-set contraction. It should be noted that in the studies given to generalize the Darbo's fixed point theorem, the sum of the coefficients of the terms on the right side of the linear inequalities used instead of the k-set contraction condition is less than 1. There is also a similar restriction for functions that correspond to the coefficients in nonlinear inequalities. As can be seen, there is no such restriction in the definition of (k, L)-set contraction for L.

Now, we present our first main result.

Theorem 2.2. *Let E be nonempty, bounded, closed and convex subset of a Banach space $\mathcal{X}$, and let $\mathcal{F}: E \to E$ be a continuous and (k, L)-set contraction. Then, $\mathcal{F}$ has a fixed point in E.*

Proof. Set $E_0 = E$ and construct a sequences of subsets E_n as

$$E_n = \overline{co}(\mathcal{F}E_{n-1}) \tag{3}$$

for $n = 1, 2, \cdots$. In this case it can be easily seen that $\mathcal{F}E_n \subseteq E_n$ and $E_{n+1} \subseteq E_n$ for all $n \in \mathbb{N}$. Indeed, by (3) we get

$$E_1 = \overline{co}(\mathcal{F}E_0) \subseteq \overline{co}(E_0) = E_0.$$

Also we get

$$\mathcal{F}E_1 \subseteq \mathcal{F}E_0 \subseteq \overline{co}(\mathcal{F}E_0) = E_1.$$

Similarly, by (3) we get

$$E_2 = \overline{co}(\mathcal{F}E_1) \subseteq \overline{co}(E_1) = E_1$$

and

$$\mathcal{F}E_2 \subseteq \mathcal{F}E_1 = \overline{co}(\mathcal{F}E_1) = E_2.$$

Continuining this way we get $\mathcal{F}E_n \subseteq E_n$ and $E_{n+1} \subseteq E_n$ for all $n \in \mathbb{N}$.

Now, if there exists a natural number n_0 such that $\mu(E_{n_0}) = 0$, then E_{n_0} is compact. In this case Theorem 1.1 implies that $\mathcal{F}$ has a fixed point in E. Next, we assume that $\mu(E_n) > 0$ for $n = 1, 2, \cdots$. By our assumptions, we get

$$\begin{aligned} \mu(E_{n+1}) &= \mu(\overline{co}(\mathcal{F}(E_n))) \\ &= \mu(\mathcal{F}(E_n)) \\ &\leq k\mu(E_n) + L\left[\mu(E_n \cup \mathcal{F}E_n) - \mu(E_n \cap \mathcal{F}E_n)\right] \\ &= k\mu(E_n) + L\left[\mu(E_n) - \mu(\mathcal{F}E_n)\right] \\ &= k\mu(E_n) + L\left[\mu(E_n) - \mu(E_{n+1})\right]. \end{aligned}$$

Hence, we have

$$(1 + L)\,\mu(E_{n+1}) \leq (k + L)\mu(E_n)$$

and so

$$\mu(E_{n+1}) \leq \frac{k + L}{1 + L}\mu(E_n).$$

Therefore, we obtain

$$\mu(E_{n+1}) \leq \lambda^{n+1}\mu(E_0)$$

for all $n \in \mathbb{N}$, where $\lambda = \frac{k+L}{1+L} < 1$ and so from the last inequality we have

$$\lim_{n\to\infty}\mu(E_n) = 0.$$

By Generalized Cantor intersection property, we get E_∞ is a nonempty and compact, where

$$E_\infty = \bigcap_{n=1}^{\infty} E_n.$$

Since $\mathcal{F}E_\infty \subseteq E_\infty$, then by Theorem 1.1, $\mathcal{F}$ has a fixed point in E. □

By taking into account of some properties of measure of noncompactness, we can describe the following concept which stronger than (k, L)-set contraction.

Definition 2.3. *Let E be a nonempty subset of a Banach space $\mathcal{X}$. A mapping $\mathcal{F} : E \to E$ is said to be a strong (k, L)-set contraction if, for each $D \subseteq E$ with bounded, $\mathcal{F}D$ is bounded and there exist $k \in [0, 1)$ and $L \geq 0$ such that*

$$\mu(\mathcal{F}D) \leq k\mu(D) + L\left[\mu(D \cup \mathcal{F}D) - \min\left\{\mu(D), \mu(\mathcal{F}D)\right\}\right]. \tag{4}$$

It is clear that every strong (k, L)-set contraction is also (k, L)-set contraction. Therefore, the proof of the following theorem is obvious.

Theorem 2.4. *Let E be nonempty, bounded, closed and convex subset of a Banach space $\mathcal{X}$, and let $\mathcal{F} : E \to E$ be a continuous and strong (k, L)-set contraction. Then, $\mathcal{F}$ has a fixed point in E.*

Now, we provide the nonlinear Leray-Schauder type alternative of Theorem 2.4.

Theorem 2.5. *Let $\mathcal{X}$ be a Banach space, E be a nonempty, bounded, closed and convex subset of $\mathcal{X}$, G an open subset of E and $\xi_0 \in G$. Suppose $\mathcal{F} : \overline{G} \to E$ be a continuous, strong (k, L)-set contraction and satisfies*

$$\mu\left(\mathcal{F}\left(D \cap \overline{G}\right)\right) \leq \mu\left(D \cap \overline{G}\right)$$

for all $D \subseteq E$. Then, either

(i) $\mathcal{F}$ has a fixed point in $\overline{G}$, or

(ii) there exists $\xi \in \partial G$ and $\lambda \in (0, 1)$ such that $\xi = \lambda\mathcal{F}\xi + (1 - \lambda)\xi_0$.

Proof. Assume (ii) does not hold and $\mathcal{F}$ has no fixed point in ∂G. Then

$$\xi \neq \lambda\mathcal{F}\xi + (1 - \lambda)\xi_0$$

for $\xi \in \partial G$ and $\lambda \in [0, 1]$. Consider the set

$$K = \left\{\xi \in \overline{G} : \xi = \lambda\mathcal{F}\xi + (1 - \lambda)\xi_0 \text{ for some } \lambda \in [0, 1]\right\}.$$

Since $\xi_0 \in K$, then K is nonempty. Also K is closed because of the continuity of $\mathcal{F}$. Further, we have $K \cap \partial G = \emptyset$. Thus there exists a continuous function $\lambda : \overline{G} \to [0, 1]$ such that $\lambda(K) = 1$ and $\lambda(\partial G) = 0$. Now, define a map $\mathcal{S} : E \to E$ as

$$\mathcal{S}\xi = \begin{cases} \lambda(\xi)\mathcal{F}\xi + (1 - \lambda(\xi))\xi_0 & , \quad \xi \in \overline{G} \\ \xi_0 & , \quad \xi \in E \backslash \overline{G} \end{cases}.$$

Then, $\mathcal{S}$ is continuous. Now, let $D \subseteq E$ be any set. Then, we have

$$\mathcal{S}(D) \subseteq \overline{co}(\mathcal{F}(D \cap \overline{G}) \cup \{\xi_0\})$$

and hence

$$\begin{aligned}
\mu(\mathcal{S}(D)) &\leq \mu(\overline{co}(\mathcal{F}(D \cap \overline{G}) \cup \{\xi_0\})) \\
&= \mu(\mathcal{F}(D \cap \overline{G})) \\
&\leq k\mu(D \cap \overline{G}) + L\left[\mu\left\{(D \cap \overline{G}) \cup \mathcal{F}(D \cap \overline{G})\right\} - \min\left\{\mu\left(D \cap \overline{G}\right), \mu\left(\mathcal{F}(D \cap \overline{G})\right)\right\}\right] \\
&\leq k\mu(D) + L\left[\mu\left(D \cap \overline{G}\right) - \mu\left(\mathcal{F}(D \cap \overline{G})\right)\right]
\end{aligned}$$

$$\leq k\mu(D) + L\left[\mu(D) - \mu(\mathcal{S}(D))\right]$$
$$\leq k\mu(D) + L\left[\mu(D \cup \mathcal{S}(D)) - \min\{\mu(D), \mu(\mathcal{S}(D))\}\right].$$

Consequently, $\mathcal{S} : E \to E$ is continuous and strong (k, L)-set contraction. Therefore, by Theorem 2.4, there exists $z \in E$ such that $z = \mathcal{S}z$. Notice that $z \in \overline{G}$ since $\xi_0 \in G$. Hence

$$z = \lambda(z)\mathcal{F}z + (1 - \lambda(z))\xi_0$$

and so $z \in K$. It follows that $\lambda(z) = 1$ which implies $z = \mathcal{F}z$. □

3. Application

This section is devoted to present a new application by aid of the Theorem 2.2. In this section, we will study the existence of solution of the following integral equation:

$$\xi(t) = a(t) + f(t, \xi(t)) \int_0^t u(s, \xi(s))ds, \tag{5}$$

where $a : [0, 1] \to \mathbb{R}$ and $f, u : [0, 1] \times \mathbb{R} \to \mathbb{R}$ are continuous functions. Consider the space

$$C[0, 1] = \{u : u : [0, 1] \to \mathbb{R} \text{ is continuous}\}$$

endowed with the norm

$$\|\xi\|_\infty = \sup\{|\xi(t)| : t \in [0, 1]\}.$$

For simplicity, denote $I = [0, 1]$ and $C[I] = C[0, 1]$.

We will now briefly summarize the definition of the measure of noncompactness in $C[I]$ that we'll be using. That measure was presented and studied in [6]. In order to do this, let us fix a nonempty, bounded subset $\mathcal{X}$ of $C[I]$. For $\xi \in \mathcal{X}$ and $\varepsilon > 0$ denoted by $\omega(\xi, \varepsilon)$, the modulus of continuity of the function ξ:

$$\omega(\xi, \varepsilon) = \sup\{|\xi(t) - \xi(s)| : t, s \in I, |t - s| \leq \varepsilon\}.$$

Moreover, let us define

$$\omega(\mathcal{X}, \varepsilon) = \sup\{\omega(\xi, \varepsilon) : \xi \in \mathcal{X}\}$$
$$\omega_0(\mathcal{X}) = \lim_{\varepsilon \to 0} \omega(\mathcal{X}, \varepsilon).$$

It can be shown that the function ω_0 is a measure of noncompactness in $C[I]$, see [6].

Define an operator $\mathcal{F} : C[I] \to C[I]$ by

$$\mathcal{F}\xi(t) = a(t) + f(t, \xi(t)) \int_0^t u(s, \xi(s))ds.$$

Hence, if ξ^* is a fixed point of $\mathcal{F}$, then it is a solution of (5).

Now consider the below conditions:

(A1) there exists $\alpha > 0$ such that

$$|f(t, \xi) - f(t, \zeta)| \leq \alpha|\xi - \zeta|$$

for all $t \in I$ and $\xi, \zeta \in \mathbb{R}$, (A2) $u(t,0)=0$ and there exists a nondecreasing function $\varphi : I \to [0, \frac{1}{\alpha})$ such that

$$|u(t,\xi)-u(t,\zeta)| \leq \varphi(|\xi-\zeta|)$$

for all $t \in I$ and $\xi, \zeta \in \mathbb{R}$,

Theorem 3.1. *In addition to (A1)-(A2), suppose that the inequality*

$$M + [\alpha r + N]\varphi(r) \leq r \tag{6}$$

has a positive solution, where

$$M = \sup\{|a(t)| : t \in I\},$$

and

$$N = \sup\{\left|f(t,0)\right| : t \in I\},$$

then the equation (5) has solution in $C[I]$.

Proof. Let r_0 is a positive solution of (6) and $B_{r_0} = \{\xi \in C[I] : \|\xi\|| < r_0\}$. First, we show that $\mathcal{F}$ maps B_{r_0} from itself. Indeed, let $\xi \in B_{r_0}$, then we have

$$\begin{aligned}
|\mathcal{F}\xi(t)| &= \left|a(t) + f(t,\xi(t)) \int_0^t u(s,\xi(s))ds\right| \\
&\leq |a(t)| + \left|f(t,\xi(t))\right| \int_0^t |u(s,\xi(s))|\, ds \\
&\leq M + \left[|f(t,\xi(t)) - f(t,0)| + \left|f(t,0)\right|\right] \int_0^t \left|\varphi(\xi(s))\right| ds \\
&\leq M + [\alpha\, |\xi(t)| + N]\, \varphi(\|\xi\|_\infty)\, t \\
&\leq M + [\alpha\, \|\xi\|_\infty + N]\, \varphi(\|\xi\|_\infty)
\end{aligned}$$

for all $t \in I$. Therefore, we have

$$\|\mathcal{F}\xi\|_\infty \leq M + [\alpha\, \|\xi\|_\infty + N]\, \varphi(\|\xi\|_\infty)$$

for all $\xi \in B_{r_0}$. Since r_0 is a positive solution of the inequality

$$M + [\alpha r + N]\varphi(r) \leq r,$$

then we get $\mathcal{F}\xi \in B_{r_0}$. Now, we show that $\mathcal{F}$ is continuous on B_{r_0}.

Let $\varepsilon > 0$ and $\xi, \zeta \in B_{r_0}$ with $\|\xi - \zeta\|_\infty \leq \varepsilon$, then we have, for all $t \in I$,

$$\begin{aligned}
|\mathcal{F}\xi(t) - \mathcal{F}\zeta(t)| &= \left|f(t,\xi(t)) \int_0^t u(s,\xi(s))ds - f(t,\zeta(t)) \int_0^t u(s,\zeta(s))ds\right| \\
&\leq \left|f(t,\xi(t)) \int_0^t u(s,\xi(s))ds - f(t,\zeta(t)) \int_0^t u(s,\xi(s))ds\right|
\end{aligned}$$

$$+\left|f(t,\zeta(t))\int_0^t u(s,\xi(s))ds - f(t,\zeta(t))\int_0^t u(s,\zeta(s))ds\right|$$

$$\leq \left|f(t,\xi(t)) - f(t,\zeta(t))\right|\int_0^t |u(s,\xi(s))|\, ds$$

$$+\left|f(t,\zeta(t))\right|\int_0^t |u(s,\xi(s)) - u(s,\zeta(s))|\, ds$$

$$\leq \alpha|\xi(t)-\zeta(t)|\varphi\left(\|\xi\|_\infty\right) + \left|f(t,\zeta(t))\right|\int_0^t |u(s,\xi(s)) - u(s,\zeta(s))|\, ds$$

$$\leq \alpha\,\|\xi-\zeta\|_\infty\,\varphi\left(r_0\right) + (\alpha r_0 + N)\int_0^t \beta(\varepsilon, r_0)ds$$

$$\leq \alpha\,\|\xi-\zeta\|_\infty\,\varphi\left(r_0\right) + (\alpha r_0 + N)\,\beta(\varepsilon, r_0),$$

where $\beta(\varepsilon, r_0)$ is defined by

$$\beta(\varepsilon, r_0) = \sup\{|u(s,\xi(s)) - u(s,\zeta(s))| : s \in I, \xi, \zeta \in B_{r_0} \text{ with } \|\xi-\zeta\|_\infty \leq \varepsilon\}.$$

Hence, we have

$$\|\mathcal{F}\xi - \mathcal{F}\zeta\|_\infty \leq \alpha\,\|\xi-\zeta\|_\infty\,\varphi\left(r_0\right) + (\alpha r_0 + N)\,\beta(\varepsilon, r_0)$$

and so $\mathcal{F}$ is continuous on B_{r_0}.

Finally, we show that $\mathcal{F}$ is (k, L)-set contraction. Let D be a nonempty subset of B_{r_0}, $\xi \in D$, $\varepsilon > 0$ and $t, s \in I$ with $|t-s| \leq \varepsilon$ and $s \leq t$, then we have

$$|\mathcal{F}\xi(t) - \mathcal{F}\xi(s)| \leq \left|a(t) + f(t,\xi(t))\int_0^t u(\tau,\xi(\tau))d\tau - a(s) - f(s,\xi(s))\int_0^s u(\tau,\xi(\tau))d\tau\right|$$

$$\leq |a(t)-a(s)| + \left|f(t,\xi(t))\int_0^t u(\tau,\xi(\tau))d\tau - f(s,\xi(s))\int_0^t u(\tau,\xi(\tau))d\tau\right|$$

$$+\left|f(s,\xi(s))\int_0^t u(\tau,\xi(\tau))d\tau - f(s,\xi(s))\int_0^s u(\tau,\xi(\tau))d\tau\right|$$

$$\leq |a(t)-a(s)| + \left|f(t,\xi(t)) - f(s,\xi(s))\right|\int_0^t |u(\tau,\xi(\tau))|\, d\tau$$

$$+\left|f(s,\xi(s))\right|\left|\int_0^t u(\tau,\xi(\tau))d\tau - \int_0^s u(\tau,\xi(\tau))d\tau\right|$$

$$\leq |a(t)-a(s)| + \left|f(t,\xi(t)) - f(s,\xi(t))\right|\varphi\left(\|\xi\|_\infty\right)$$

$$+\left|f(s,\xi(t)) - f(s,\xi(s))\right|\varphi\left(\|\xi\|_\infty\right)$$

$$+\left|f(s,\xi(s))\right|\left|\int_s^t u(\tau,\xi(\tau))d\tau\right|$$

$$\leq \omega(a,\varepsilon) + \omega_f(r_0,\varepsilon)\varphi(r_0) + \alpha|\xi(t)-\xi(s)|\varphi(r_0) + (\alpha r_0 + N)\varphi(r_0)\varepsilon,$$

where

$$\omega_f(r_0,\varepsilon) = \sup\{\left|(f(t,\xi)-f(s,\xi))\right| : t,s\in[0,1],\ |t-s|\leq\varepsilon,\ \xi\in[-r_0,r_0]\}.$$

Hence, we have the estimate

$$\omega(\mathcal{F}\xi,\varepsilon) \leq \omega(a,\varepsilon) + \omega_f(r_0,\varepsilon)\varphi(r_0) + \alpha\omega(\xi,\varepsilon)\varphi(r_0) + (\alpha r_0+N)\varphi(r_0)\varepsilon$$

and so taking $\varepsilon \to 0$ we get

$$\begin{aligned}\omega_0(\mathcal{F}D) &\leq k\omega_0(D)\\ &\leq k\omega_0(D) + L\left[\omega_0(D\cup\mathcal{F}D) - \omega_0(D\cap\mathcal{F}D)\right],\end{aligned}$$

where $k = \alpha\varphi(r_0) < 1$. Therefore, $\mathcal{F}$ is (k,L)-set contraction for every $L \geq 0$. Finally, combining the above estimate and all properties of the operator $\mathcal{F}$ given before and by using Theorem 2.2, the equation (5) has at least one solution in $C[I]$. □

References

[1] A. Aghajani, M. Mursaleen, A. S. Haghighi, *Fixed point theorems for Meir-Keeler condensing operators via measure of noncompactness*, Acta Mathematica Scientia, **35** (3) (2015), 552–566.

[2] A. Aghajani, R.Allahyari, M. Mursaleen, *A generalization of Darbo's theorem with application to the solvability of systems of integral equations*, Journal of Computational and Applied Mathematics, **260** (2014), 68–77.

[3] A. Aghajani, E. Pourhadi, J. Trujillo, *Application of measure of noncompactness to a Cauchy problem for fractional differential equations in Banach spaces*, Fractional Calculus and Applied Analysis, **16** (4)(2013), 962–977.

[4] R. Arab, M. Mursaleen, S. M. H Rizvi, *Positive solution of a quadratic integral equation using generalization of Darbo's fixed point theorem*, Numerical Functional Analysis and Optimization, **40** (10) (2019), 1150–1168.

[5] R. Arab, H. K. Nashine, N. H. Can, T. T. Binh, *Solvability of functional-integral equations (fractional order) using measure of noncompactness*, Advances in Difference Equations **2020** (1) (2020), 1–13.

[6] J. Banaś, K. Goebel, *Measures of noncompactness in Banach spaces*, Lecture Notes in Pure and Applied Mathematics, vol. 60, M. Dekker New York-Basel, 1980.

[7] J. Banaś, B. Rzepka, *An application of a measure of noncompactness in the study of asymptotic stability*, Applied Mathematics Letters, **16** (1) (2003), 1–6.

[8] J. Banaś, J. Caballero, J. Rocha, K. Sadarangani, *Monotonic solutions of a class of quadratic integral equations of Volterra type*, Computers & Mathematics with Applications, **49** (5-6) (2005), 943–952.

[9] S. Beloul, M. Mursaleen, A. H. Ansari, *A generalization of Darbo's fixed point theorem with an application to fractional integral equations*, Journal of Mathematical Inequalities **15**(3) (2021), 911–921.

[10] F. E. Browder, *A further generalization of the Schauder fixed point theorem*, Duke Mathematical Journal, **32** (4) (1965), 575–578.

[11] F. E. Browder, *A new generalization of the Schauder fixed point theorem*, Mathematische Annalen, **174**(4) (1967), 285–290.

[12] Ü. Çakan, İ. Özdemir, *An application of Darbo fixed-point theorem to a class of functional integral equations*, Numerical Functional Analysis and Optimization, **36** (1) (2015), 29–40.

[13] G. Darbo, *Punti uniti in trasformazioni a codominio non compatto*, Rendiconti del Seminario Matematico della Università di Padova, **24** (1955), 84–92.

[14] A. Granas, J. Dugundji, *Fixed point theory*, vol. 14, Springer, 2003.

[15] B. Hazarika, R. Arab, M. Mursaleen, *Applications of measure of noncompactness and operator type contraction for existence of solution of functional integral equations*, Complex Analysis and Operator Theory **13** (2019), 3837–3851.

[16] K. Kuratowski, *Sur les espaces complets*, Fundamenta Mathematicae **1** (15) (1930), 301–309.

[17] M. Mursaleen, R. Arab, *On existence of solution of a class of quadratic-integral equations using contraction defined by simulation functions and measure of noncompactness*, Carpathian Journal of Mathematics, **34** (3) (2018), 371–78.

[18] M. Mursaleen, V. Rakočević, *A survey on measures of noncompactness with some applications in infinite systems of differential equations*, Aequationes Mathematicae, (2021), 1–26.

[19] M. Mursaleen, S. Rizvi, *Solvability of infinite systems of second order differential equations in c_0 and ℓ_1 by Meir-Keeler condensing operators*, Proceedings of the American Mathematical Society, **144**(10) (2016), 4279–4289.

[20] H. K. Nashine, R. W. Ibrahim, R. Arab, M. Rabbani, *Generalization of Darbo-type fixed point theorem and applications to integral equations*, Advances in Metric Fixed Point Theory and Applications, Springer, 2021, pp. 333–364.

[21] J. Schauder, *Der fixpunktsatz in funktionalraümen*, Studia Mathematica, **2** (1) (1930), 171–180.

[22] J. H. Shapiro, *The Schauder fixed-point theorem*, A Fixed-Point Farrago, Springer, 2016, pp. 75–81.

[23] E. Zeidler, *Nonlinear functional analysis and it's applications I: Fixed-Point Theorems*, (1986), 897.