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Fibonacci f-statistical convergence and Korovkin type approximation theorems

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Abstract. This article presents the notion of Fibonacci statistical convergence using density by moduli, this indeed extends the concept of Fibonacci statistical convergence which we call as Fibonacci f-statistical convergence. The primary focus of this study is to prove the Korovkin type theorem of Boyanov and Veselinov [5] using Fibonacci f-statistical convergence. A connection between Fibonacci statistical convergence and Fibonacci f-statistical convergence is also established.

1. Introduction

Statistical convergence is a concept in mathematical analysis that provides a way to measure the convergence of a sequence of numbers based on their statistical properties. Unlike traditional notions of convergence, which focus on the limit of individual terms in the sequence, statistical convergence considers the behavior of the sequence as a whole. More formally, a sequence of real numbers (x_n) converges statistically to a real number L if, for every $\epsilon > 0$, $\lim_{n\to\infty} \frac{1}{n}\{k \leq n \ : \ |x_k - L| \geq \epsilon\} = 0$. This means that the proportion of terms in the sequence that are far away from L is small in some precise sense. Statistical convergence was first introduced independently by Fast [7] and Steinhaus [16] in 1951 and has since been studied extensively in the context of various mathematical fields, including real analysis, functional analysis, probability theory, etc. Later, Mursaleen [14] introduced the idea of λ-statistical convergence, which is a further extension of statistical convergence. Aizpuru et al. [1] generalised the concept of statistical convergence by using the density by modulli known as f-statistical convergence. Bhardwaj and Dhawan [4] extended this convergence to f-statistical convergence of order α, and studied this concept in summability. Later, they proved the Korovkin type theorems by using the concept of f-statistical convergence and f-lacunary statistical convergence [2, 3]. Boyanov and Veselinov [5] proved a Korovkin type theorem using the test functions $\{1, e^{-x}, e^{-2x}\}$ in the Banach space $C^*[0,\infty)$, which consists of continuous functions on the interval $[0,\infty)$. The space guarantees the existence of the limit of $f(x)$ as x approaches infinity, measured by the uniform norm $\|.\|_\infty$. Kizmaz [12] first introduced the idea of difference sequence spaces, defined as $X(\Delta) = \{x = (x_n) \in w : (x_k - x_{k-1}) \in X\}$, for $X = \ell_\infty, c$ and c_0. Later, Kara [13] generalized the above difference sequence spaces by using Fibonacci sequences denoted by $X(F)$ and defined as

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$X(F) = \{x = (x_n) \in w : Fx_n \in X\}$. Here, $F = (\tilde{f_{nk}})$ is a Fibonacci infinite matrix and f_n denotes the nth Fibonacci number given by

$$\tilde{f_{nk}} = \begin{cases} -\frac{f_{n+1}}{f_n}, & \text{if } k = n-1, \\ \frac{f_n}{f_{n+1}}, & \text{if } k = n, \\ 0, & \text{if } 0 \le k \le n-1 \text{ or } k > n. \end{cases} \tag{1}$$

The corresponding F-transform of a sequence $x = (x_n)$ is given by

$$Fx_n = \begin{cases} \frac{f_0}{f_1} x_0 = x_0, & \text{if } n = 0, \\ \frac{f_n}{f_{n+1}} x_n - \frac{f_{n+1}}{f_n} x_{n-1}, & \text{if } n \ge 1. \end{cases} \tag{2}$$

Then, Kirisci and Karaisa [11] studied Fibonacci difference sequence spaces and introduced the concept of Fibonacci statistical convergence. They also proved the second Korovkin type theorem using this convergence criterion.

It is striking to recall that an unbounded modulus function f is a mapping from $[0, \infty)$ to $[0, \infty)$ such that $f(x) = 0$ if and only if $x = 0$, $f(x + y) \le f(x) + f(y)$ for $x \ge 0,\ y \ge 0$, f is increasing and f is continuous from right at 0. We use the unbounded modulus function to generalize the idea of Fibonacci statistical convergence, which we call as Fibonacci f-statistical convergence. In this study, we establish some results on Fibonacci f-statistical convergence and use those to prove a Korovkin type theorem of Boyanov and Veselinov [5].

We recall certain definitions that will be relevant to our discussion.

Definition 1.1. *[15] A matrix $A = (a_{mn})$ is classified as regular when it meets the following conditions:*

1. *For every m, $\sum_{n=1}^{\infty} |a_{mn}| \le K$, where K is a constant.*
2. *For all n, $\lim_{m\to\infty} a_{mn} = 0$.*
3. $\lim_{n\to\infty} \sum_{n=1}^{\infty} a_{mn} = 1$.

Definition 1.2. *[8] A sequence (x_n) is statistical bounded if there exist a number $M > 0$, such that* $\lim_{n\to\infty} \dfrac{1}{n} |\{k \in \mathbb{N} : |x_n| > M\}| = 0$.

Remark 1.3. *Every statistically convergent sequence is statistically bounded.*

Definition 1.4. *[1] Let f be an unbounded modulus function. The f-density of $K \subset \mathbb{N}$, denoted as $D^f(K)$, is defined by*

$$D^f(K) = \lim_{n\to\infty} \frac{f(|\{k \le n : k \in K\}|)}{f(n)}$$

provided the limit exist.

Definition 1.5. *[2] Let f be an unbounded modulus function and X be a normed space then the sequence (x_n) in X is said to be f-statistically convergent to L, if for every $\epsilon > 0$, $D^f\left(\{k \in \mathbb{N} : \|x_n - L\| \ge \epsilon\}\right) = 0$.*

2. Some remarks on Fibonacci statistical convergence

Kirisci and Karaisa [11] were the first to introduce the concept of Fibonacci statistical convergence. They defined this convergence in the following manner. A sequence (x_n) is said to be Fibonacci statistical convergent to L if for any given $\epsilon > 0$, $\lim_{n\to\infty} \frac{1}{n} |\{k \le n : |Fx_k - L| \ge \epsilon\}| = 0$. They also proved a Korovkin type theorem in the context of Fibonacci statistical sense. For our convenience we denote the Fibonacci statistical limit as $F - st \lim_n x_n = L$. Here, $F = (\tilde{f_{nk}})$ is an infinite matrix defined by

$$\tilde{f_{nk}} = \begin{cases} \frac{f_{n+1}}{f_n}, & \text{if } k = n-1, \\ -\frac{f_n}{f_{n+1}}, & \text{if } k = n, \\ 0, & \text{if } 0 \leq k \leq n-1 \text{ or } k > n, \end{cases} \tag{3}$$

where f_n denotes the nth Fibonacci number. The F-transform of the sequence (x_n) is given by

$$Fx_n = \begin{cases} \frac{f_0}{f_1} x_0 = x_0, & \text{if } n = 0, \\ \frac{f_{n+1}}{f_n} x_{n-1} - \frac{f_n}{f_{n+1}} x_n, & \text{if } n \geq 1. \end{cases} \tag{4}$$

The matrix given in (1) is non-regular. For our study, we have obtained (3) and (4) by modifying (1) to ensure the regularity of the matrix. From now onwards we use this revised F-transform throughout this paper. We reviewed the paper of Kirisci and Karaisa [11] and identified some errors in the proofs of certain theorems.

The following result is a corrected version of Theorem 3.6 of [11].

Theorem 2.1. *Let $x = (x_n)$ be a sequence for which there is a F-statistically convergent sequence $y = (y_n)$ such that $Fx_k = Fy_k$ for almost all k, then $x = (x_n)$ is a F-statistically convergent sequence.*

Proof. Suppose that $Fx_k = Fy_k$ for almost all k and $F - st \lim y_k = L$. Then for each $\epsilon > 0$ and for all n $\{k \leq n : |Fx_k - L| \geq \epsilon\} \subset \{k \leq n : Fx_k \neq Fy_k\} \cup \{k \leq n : |Fy_k - L| \geq \epsilon\}$. Therefore,

$$\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : |Fx_k - L| \geq \epsilon\}| \leq \lim_{n\to\infty} \frac{1}{n} \left|\{k \leq n : Fx_k \neq Fy_k\}\right| + \lim_{n\to\infty} \frac{1}{n} \left|\{k \leq n : |F_{y_k} - L| \geq \epsilon\}\right|.$$

By looking into the two sets on the right side of the inequality, we can observe that their natural density is zero. As a result, it is straightforward to verify that $\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : |Fx_k - L| \geq \epsilon\}| = 0$. Hence ends the proof. □

Remark 2.2. *In Theorem 3.6 of [11], line no. 2, the right side of the following $\{k \in \mathbb{N} : |Fx_k - L| \geq \epsilon\} \subset \{k \in \mathbb{N} : Fx_k \neq Fy_k\} \cup \{k \in \mathbb{N} : |Fx_k - L| \leq \epsilon\}$ cannot have a natural density zero because the natural density of the set $\{k \in \mathbb{N} : |Fx_k - L| \leq \epsilon\}$ is nonzero. If this density were zero, it would imply that the sequence (x_k) is not Fibonacci statistically converge to L.*

We observed that, in Theorem 3.17 of [11], an error occurred in the proof. On page 11, line no. 7, after stating the inequalities, the linear operator $L_k(.,x)$ was replaced with $T_k(.,x) = FL_k(.,x)$. This substitution is problematic because the sequence (L_k) consists of positive linear operators, whereas $F(x_k) = \frac{f_n}{f_{n+1}} x_k - \frac{f_{k+1}}{f_k} x_{k-1}$ is not always positive. Consequently, the inequality may not hold true after such a replacement. To illustrate, consider the sequence (L_n) of positive linear operators, where $L_n(k,x) = k$, $k = 1, 2, 3...$. It satisfies $\lim_n \|L_n(5,x) - 3\|_\infty \leq \lim_n \|L_n(4,x) - 1\|_\infty$. However, if we substitute $L_n(.,x)$ with $FL_n(.,x)$, i.e., $\lim_n \|FL_n(5,x) - 3\|_\infty \nleq \lim_n \|FL_n(4,x) - 1\|_\infty$, the left side of the inequality evaluates to 8, while the right side becomes 5. This discrepancy arises from the fact that the matrix defined in (1) is not regular, as $\lim_{n\to\infty} \left(\frac{f_n}{f_{n+1}} - \frac{f_{n+1}}{n}\right) = -1$. For this, we replaced the infinite matrix specified in (1) with (3) and the F-transform specified in (2) is replaced with (4). We present the following notations that are useful for formulating the Korovkin-type theorem. Let $C_{2\pi}(\mathbb{R})$ be the set of all functions that are 2π-periodic over the real numbers $\mathbb{R}$, and these functions are evaluated using the norm $\|f\|_\infty = \sup_{x\in\mathbb{R}} |f(x)|$. The Banach space $C^*[0,\infty)$ comprises continuous functions defined over the interval $[0,\infty)$ that possess a finite limit as x approaches infinity. The following result represents a corrected version of the proof of the Korovkin-type theorem as presented in [11].

Theorem 2.3. *Let (L_n) be the sequence of positive linear operators mapping from $C_{2\pi}(\mathbb{R})$ into $C_{2\pi}(\mathbb{R})$. Then, for all $f \in C_{2\pi}(R)$,*

$$F - st \lim_{n\to\infty} \|L_n(f, x) - f(x)\|_\infty = 0, \tag{5}$$

if and only if

$$F - st \lim_{n\to\infty} \|L_n(f_i, x) - f_i(x)\|_\infty = 0, \quad i = 0, 1, 2, \tag{6}$$

where $f_0(x) = 1$, $f_1(x) = \cos x$ and $f_2(x) = \sin x$.

Proof. The proof follows the same pattern as Theorem 3.18 stated in [11], as long as the F-transformed is substituted with the revised F-transform mentioned in (2). □

The following is a new result that establishes a relationship between statistical convergence and Fibonacci statistical convergence.

Theorem 2.4. *Every statistically convergent sequence is Fibonacci statistically convergent.*

Proof. Let a sequence (x_n) statistically converges to a limit L. Using Theorem 1 stated in the reference [9], we can express this as $\lim_{n\in\mathbb{N}/K} x_n = L$, where K represents a subset of natural numbers with zero natural density. The matrix defined in (3) is considered regular, enabling us to establish $\lim_{n\in\mathbb{N}/K} Fx_n = L$, where the natural density of set K is zero. Consequently, $\lim_{n\to\infty} \frac{1}{n}|\{k \le n : |Fx_k - L| \ge \epsilon\}| = 0$. Hence ends the proof. □

Remark 2.5. *The converse of Theorem 2.4 is false. In order to show that, we consider the sequence $\{f_{n+1}^2\}_{n=1}^{\infty}$, which is F-statistically convergent to zero, i.e., $F(f_{n+1}^2) = 0$. However, it is not statistically bounded, indicating that the sequence is not statistically convergent.*

3. Fibonacci f-statistical convergence

The objective of this section is to introduce the concept of Fibonacci f-statistical convergence, to establish some related results, and to prove a Korovkin type theorem within this framework.

Definition 3.1. *Let f be an unbounded modulus function. A sequence (x_n) is said to be convergent to L in Fibonacci f-statistical sense, denoted as $F^f - st \lim_n x_n = L$, if for all $\epsilon > 0$,*

$$\lim_{n\to\infty} \frac{1}{f(n)} f\left(|\{k \le n : |Fx_k - L| \ge \epsilon\}|\right) = 0.$$

It is to note that, every Fibonacci f-statistical convergent sequence is Fibonacci statistically convergent, but the converse may not be true. The following theorems establish this fact.

Theorem 3.2. *Every sequence that converges in the Fibonacci f-statistical sense also converges in the Fibonacci statistical sense.*

Proof. Suppose that a sequence (x_n) converges to L in the Fibonacci f-statistical sense. Then, for a given $p \in \mathbb{N}$, there exists n such that $p \ge n$, and we have

$$\frac{1}{f(n)} f\left(|k \le n : |Fx_k - L| \ge \epsilon|\right) \le \frac{1}{p},$$

where f is an unbounded modulus function. By the subadditive property of f, we write

$$f\left(|k \le n : |Fx_k - L| \ge \epsilon|\right) \le \frac{1}{p} p f(\frac{n}{p}).$$

As the function f is increasing, we can have

$$\{|k \leq n : |Fx_k - L| \geq \epsilon|\} \leq \frac{n}{p}.$$

Now, $p \to \infty$ when n goes to ∞, we have

$$\lim_{n\to\infty} \frac{1}{n}\{|k \leq n : |Fx_k - L| \geq \epsilon|\} = 0.$$

Hence ends the proof. □

Definition 3.3. *A sequence (x_n) is said to be Fibonacci f-statistical Cauchy sequence if for all $\epsilon > 0$ and there exist a number $N(\epsilon)$ such that*

$$\frac{1}{f(n)} f\left(\{k \leq n : |Fx_k - Fx_N| \geq \epsilon|\}\right) = 0.$$

Theorem 3.4. *Every Fibonacci f-statistical convergent sequence is Fibonacci f-statistical Cauchy sequence.*

Proof. Let (x_n) be a Fibonacci f-statistical convergent sequence. For a given $\epsilon > 0$, choose $N(\epsilon)$ in such a way that $|Fx_N - L| < \frac{\epsilon}{2}$. Now, we can see that

$$\{k \leq n : |Fx_k - Fx_N| \geq \epsilon\} \subset \{k \leq n : |Fx_k - L| \geq \frac{\epsilon}{2}\} \cup \{k \leq n : |Fx_N - L| \geq \frac{\epsilon}{2}\}.$$

As the modulus function f is subadditive and increasing, we can write

$$\frac{1}{f(n)} f\left(|\{k \leq n : |Fx_k - Fx_N| \geq \epsilon\}|\right) \leq \frac{1}{f(n)} f\left(\left|\{k \leq n : |Fx_k - L| \geq \frac{\epsilon}{2}\}\right|\right) + \frac{1}{f(n)} f\left(\left|\{k \leq n : |Fx_N - L| \geq \frac{\epsilon}{2}\}\right|\right).$$

It is easy to check that right side of the above inequality have limit zero. Therefore,

$$\lim_{n\to\infty} \frac{1}{f(n)} f\left(|\{k \leq n : |Fx_k - Fx_N| \geq \epsilon\}|\right) = 0.$$

Hence ends the proof. □

Korovkin's first and second theorems were initially proven in statistical sense by Gadjiev, Orhan [10] and Duman [6], respectively. Boyanov and Veselinov [5] proved the Korovkin type theorems by utilizing the test functions $1, e^{-x}, e^{-2x}$ in the Banach space $C^*[0, \infty)$. Later, Bhardwaj and Dhawan [2] proved all these theorems in f-statistical sense.

The following theorem is a Korovkin type theorem due to Bayanov and Veselinov [5] in Fibonacci f-statistical sense.

Theorem 3.5. *Let f be an unbounded modulus function and (L_n) be the sequence of positive linear operators from $C^*[0, \infty)$ to itself, then it satisfies*

$$F^f - \lim_n \|L_n(g, x) - g(x)\|_\infty = 0 \tag{7}$$

if and only if

$$F^f - \lim_n \|L_n(1, x) - 1\|_\infty = 0, \tag{8}$$

$$F^f - \lim_n \|L_n(e^{-t}, x) - e^{-x}\|_\infty = 0, \tag{9}$$

$$F^f - \lim_n \|L_n(e^{-2t}, x) - e^{-2x}\|_\infty = 0. \tag{10}$$

Proof. Suppose that (7) holds, then (8) to (10) are direct consequence of (7) for some particular values of $g(x)$. Conversely, suppose that (8) to (10) hold. Let $g(x) \in C^*[0,\infty)$, then there exists $M \in \mathbb{R}$, such that

$$|g(t) - g(x)| \le 2M, \tag{11}$$

and it can be easily demonstrated that for every $\epsilon > 0$ there exist a $\delta > 0$, such that

$$|g(t) - g(x)| < \epsilon \ \text{ whenever } |e^{-t} - e^{-x}| < \delta,\ t, x \in [0,\infty). \tag{12}$$

From (11) and (12), we have

$$-\epsilon - \frac{2M}{\delta^2}(e^{-t} - e^{-x})^2 < g(t) - g(x) < \epsilon + \frac{2M}{\delta^2}(e^{-t} - e^{-x})^2.$$

As L_n is linear and monotonic, therefore

$$-\epsilon L_n(1,x) - \frac{2M}{\delta^2} L_n((e^{-t} - e^{-x})^2, x) < L_n(g,x) - g(x)L_n(1,x) < \epsilon L_n(1,x) + \frac{2M}{\delta^2} L_n((e^{-t} - e^{-x})^2, x). \tag{13}$$

On the other hand

$$L_n(g,x) - g(x) = [L_n(g,x) - g(x)L_n(1,x)] + g(x)[L_n(1,x) - 1]. \tag{14}$$

Using inequality (13) in (14), we have

$$\begin{aligned} L_n(g,x) - g(x) &\le \epsilon L_n(1,x) + \frac{2M}{\delta^2} L_n((e^{-t} - e^{-x})^2, x) + g(x)[L_n(1,x) - 1] \\ &= \epsilon L_n(1,x) + \frac{2M}{\delta^2}\left\{[L_n(e^{-2t},x) - e^{-2x}] - 2e^{-x}[L_n(t,x) - x] + e^{-2x}[L_n(1,x) - 1]\right\} \\ &\qquad + g(x)[L_n(1,x) - 1] \\ &= \left(\epsilon + \frac{2M}{\delta^2}e^{-2x} + g(x)\right)[L_n(1,x) - 1] + \frac{2M}{\delta^2}[L_n(e^{-2t},x) - e^{-2x}] - \frac{4M}{\delta^2}e^{-x}[L_n(e^{-t},x) - e^{-x}] + \epsilon. \end{aligned}$$

Let α_1, α_2, and α_3 be the supremum of $\left(\epsilon + \frac{2M}{\delta^2}e^{-2x} + g(x)\right)$, $\frac{2M}{\delta^2}$, and $\frac{4M}{\delta^2}e^{-x}$ in $[a,b]$.
Choosing $S = \max\{\alpha_1, \alpha_2, \alpha_3\}$, we obtained

$$\|L_n(g,x) - g(x)\|_\infty \le S\left\{\|L_n(1,x) - 1\|_\infty + \|L_n(e^{-2t},x) - e^{-2x}\|_\infty + \|L_n(e^{-t},x) - e^{-x}\|_\infty\right\} + \epsilon.$$

Now replacing $L_n(.,x)$ with $Q_n = FL_n(.,x)$ and for $\epsilon' > 0$, we have

$$\begin{aligned} P &= \left\{k \in \mathbb{N} : \|Q_n(1,x) - 1\|_\infty + \|Q_n(e^{-2t},x) - e^{-2x}\|_\infty + \|Q_n(e^{-t},x) - e^{-x}\|_\infty \ge \frac{\epsilon' - \epsilon}{S}\right\}, \\ P_1 &= \left\{k \in \mathbb{N} : \|Q_n(1,x) - 1\|_\infty \ge \frac{\epsilon' - \epsilon}{3S}\right\}, \\ P_2 &= \left\{k \in \mathbb{N} : \|Q_n(e^{-t},x) - e^{-x}\|_\infty \ge \frac{\epsilon' - \epsilon}{3S}\right\}, \\ P_3 &= \left\{k \in \mathbb{N} : \|Q_n(e^{-2t},x) - e^{-2x}\|_\infty \ge \frac{\epsilon' - \epsilon}{3S}\right\}. \end{aligned}$$

It is easy to verify that $P \subset P_1 \cup P_2 \cup P_3$. By utilizing the subadditive and monotonic properties of the modulus function f and multiplying both sides by $\frac{1}{f(n)}$, we can express it as follows:

$$\begin{aligned}
&\frac{1}{f(n)} f\left(\left|\{k \leq n : \|Q_n(g,x) - g(x)\|_\infty \geq \epsilon'\}\right|\right) \\
&\quad \leq \frac{1}{f(n)} f\left(\left|\left\{k \leq n : \|Q_n(1,x) - 1\|_\infty \geq \frac{\epsilon' - \epsilon}{3S}\right\}\right|\right) \\
&\quad + \frac{1}{f(n)} f\left(\left|\left\{k \leq n : \|Q_k(e^{-t},x) - e^{-x}\|_\infty \geq \frac{\epsilon' - \epsilon}{3S}\right\}\right|\right) \\
&\quad + \frac{1}{f(n)} f\left(\left|\left\{k \leq n : \|Q_n(e^{-2t},x) - e^{-2x}\|_\infty \geq \frac{\epsilon' - \epsilon}{3S}\right\}\right|\right).
\end{aligned}$$

Using (8) - (10), we have $F^f - \lim_n \|L_n(g,x) - g(x)\|_\infty = 0$. Hence ends the proof. □

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