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Some characterizations of soft continuous mappings using soft graphs

Tareq M. Al-shami[a,b,*], Sandeep Kaur[c], Alkan Özkan[d], M. Hosny[e], Abdelwaheb Mhemdi[f]

[a] *Jadara University Research Center, Jadara University, Jordan*
[b] *Department of Engineering Mathematics & Physics, Faculty of Engineering & Technology, Future University, New Cairo, Egypt*
[c] *Department of Mathematics and Statistics, Central University of Punjab, Bathinda, India*
[d] *Department of Mathematics, Faculty of Arts and Sciences, Iğdır University, Iğdır 76000, Turkey*
[e] *Department of Mathematics, College of Science, King Khalid University, Abha 61413 Saudi Arabia*
[f] *Department of Mathematics, College of Sciences and Humanities in Aflaj, Prince Sattam bin Abdulaziz University,Riyadh, Saudi Arabia*

Abstract. The aim of this study is to contribute to the theoretical studies on soft closed graphs and soft continuous mappings. We give a characterization of soft continuity using soft points and obtain a sufficient condition for the soft equalizer to be soft closed. We also present the notion of a soft filter generated by the soft net and vice-versa and prove their convergence results correspond to each other. Further, we prove that with a soft Hausdorff and soft compact co-domain of soft mapping, soft continuity is equivalent to soft mapping having a soft closed graph.

1. Introduction

The soft set theory presented by Molodtsov in [30] is a completely new approach to the modeling of vagueness, and it has a rich potential to be applied in several ways. The theory of soft set has become extremely popular and is used in various areas such as engineering, computer science, and medicine [18, 19] as well as economics and optimization theory. In [14], Aygün defined a soft matrix, which is a representation of a soft set. This representation of soft sets has several advantages and is useful for storing a soft set in computer memory. Thus, soft set theory is very useful for solving complex problems in computer sciences. Many efforts have focused on generalizing and extending soft sets, for example, it was provided bipolar soft sets [6], double-framed soft sets [10] and N-soft set [23].

Shabir and Naz [32], in particular, started to study the soft topological spaces. We draw the attention of the readers to that Çağman et al. [16] adopted another approach to study soft topological spaces. Later on, many authors have done work on the soft topology spaces in [1–3, 7, 11, 17, 25, 29]. Aygünoğlu and Aygün [15] presented a soft continuity based on soft mapping and a soft product. In [33], Sahin et al. presented the concept of the soft filter by using soft sets and investigated their related properties using the convergence theory of soft filter. Demir and Özbakir [20] introduced new concepts such as soft point, soft net, and

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* Corresponding author: Tareq M. Al-shami
Email addresses: tareqalshami83@gmail.com (Tareq M. Al-shami), sandybar86@gmail.com (Sandeep Kaur), alkan.ozkan@igdir.edu.tr (Alkan Özkan), Maly@kku.edu.sa (M. Hosny), mhemdiabd@gmail.com (Abdelwaheb Mhemdi)

convergence of soft nets, and defined soft Hausdorff spaces using soft points. In [27], the concept of soft closed graphs of soft mappings was introduced and the characterizations of soft closed graph of identity soft mapping were studied. Characterizations of soft closedness of softgraphs utilizing soft nets were also obtained. Quite recently, four types of soft separation axioms with some practical applications have been explored and discussed [21, 22].

Compact spaces are among the most significant classes of general topology spaces, and they have a wide range of properties that can be applied in many fields. This kind of space was discovered and examined in a soft setting by Zorlutuna et. al [37]. A study of soft compact spaces based on a soft filter and a maximal soft filter was carried out by Wang [35]. Hida [26] exploited the nature of belonging relations in the soft set theory to display novel sorts of compactness. Further contributions to soft compactness using extensions of soft open sets have been achieved by several researchers [12, 13]. Soft versions of nearly Menger and almost Menger spaces were studied by Al-shami and Kočinac [8, 9]. As noted from the existing literature, there are numerous divergences of properties and characterizations of topological concepts in classical and soft environments, for more details we refer the readers to [5, 34].

It is known that the closed graph theorem (For a map $f : U \to V$, where U is arbitrary topological space and V is a compact, Hausdorff space, the graph of f is closed if and only if f is continuous) is a basic result that characterizes continuous functions in terms of their graphs. The Closed Graph Theorem is a powerful tool that bridges the concepts of topology and continuity, making it a cornerstone in various branches of mathematics, including functional analysis, optimization, and mathematical physics. In functional analysis, the theorem is frequently applied to the study of sequence and net convergence. It Specifically ensures that if a sequence of points in the domain converges to a limit, then the corresponding images under the function also converge to a limit. This has consequences for understanding the behavior of functions in various contexts.

In this study, characterizations of soft continuity by using the concept of soft points are given. Also, we introduce a soft filter generated by the soft net and vice-versa and show their convergence results correspond to each other. We obtain a relationship between soft closed graphs and soft continuity in order to understand the new mathematical discipline further.

2. Preliminaries

Throughout this study, U is referred to as the first universal set, D consists of all the parameters of U, $P(U)$ denotes the power set of U, and $S(U, D)$ represents the family of all soft sets that depend on the parameter D over U.

Definition 2.1. ([30]) A soft set over U is a pair (G, D), where $G : D \to P(U)$ is a function from the set of parameters D into $P(U)$, this soft set can be expressed by: $(G, D) = \{(d, G(d)) : d \in D \text{ and } G(d) \in P(U)\}$. In other words, with D being a parametrizing set, a soft set is defined as the parametrizable family of subsets of the universe U.

Throughout the study, the G_D notation symbol will be used instead of the (G, D) notation symbol representing the soft set.

Definition 2.2. ([4, 28, 31]) A soft set G_D over U is named

(i) a soft point if there exist $d \in D$ and $x \in U$ such that $G(d) = x$ and $G(t) = \emptyset$, for each $t \in D - d$ and we write by (P_d^x). The family of all soft points in U is denoted by $SP(U, D)$. Remark that soft points form the counterpart of classical points in soft settings.

(ii) a complement of another soft set H_D if $G(t) = U - H(t)$ for each $t \in D$. We write $G_D^c = H_D$, where the notation ^{c} is refereed to the complement operation.

(iii) an absolute soft set if $G(t) = U$ for all $t \in D$.

(iv) a null soft set if its complement is an absolute soft set.

Note that the soft points $P^{x_1}_{d_1}, P^{x_2}_{d_2}$ are named distinct providing that $x_1 \neq x_2$ or $d_1 \neq d_2$. Also, it is said that $P^x_d \in G_D$ if x belongs to $G(d)$.

Definition 2.3. ([4, 28]) It was defined as the union and intersection of soft sets G_D and H_D over U as follows.

(i) $G_D \tilde{\cup} H_D = F_D$, where $F(t) = G(t) \cup H(t)$ for all $t \in D$.

(ii) $G_D \tilde{\cap} H_D = F_D$, where $F(t) = G(t) \cap H(t)$ for all $t \in D$.

Definition 2.4. ([24]) We call G_D a soft subset of H_D providing that every $G(t)$ is a subset of $H(t)$.

Definition 2.5. ([32]) Let U be a non-empty set and τ be any subset of $P(U)$. If the subset τ satisfies the following properties, the τ family is called a soft topology on U, the (U, τ, D) triplet is a soft topological space (briefly, soft space), and each member of the τ family is called an open set.

(i) $\phi, \tilde{U} \in \tau$,

(ii) if $G_D, H_D \in \tau$, then $G_D \tilde{\cap} H_D \in \tau$,

(iii) if $(G_i, D) \in \tau$, each $i \in I$, then $\tilde{\cup}(G_i, D) \in \tau$.

Definition 2.6. ([36]) A set T is said to be a directed set if there exists a relation $\geq$ on T with:

(i) $t \geq t$ for all $t \in T$.

(ii) If $t_1 \geq t_2$, $t_2 \geq t_3$ then $t_1 \geq t_3$ for all $t_1, t_2, t_3 \in T$.

(iii) If $t_1, t_2 \in T$, there exists some $t_3 \in T$ such that $t_3 \geq t_1, t_3 \geq t_2$.

Definition 2.7. ([20]) If T is a directed set, U is a non-empty set. A soft function $x : T \to SP(U, D)$ is said to be a soft net in U and is denoted by $(P^{x_t}_{d_t})_{t \in T}$.

Definition 2.8. ([27]) Let $(K, n) : S(U, D) \to S(V, E)$ be a soft mapping. Then soft graph of (K, n) is a soft set $G(K, n)_{D\times E}$, where $G(K, n) : D \times E \to \mathbf{P}(U \times V)$ is defined by

$$G(K, n)(d, e) = \begin{cases} G(K) & \text{if } e = n(d) \\ \phi & \text{if } e \neq n(d) \end{cases}$$

Here $G(K)$ is the graph of the function K.

Definition 2.9. ([27]) Let (U, τ_1, D) and (V, τ_2, E) be soft spaces, then (K, n) is a soft mapping with *soft closed graph*, if $G(K, n)_{D\times E}$ is soft closed in the soft product topological space $(U \times V, \tau_1 \times \tau_2, D \times E)$.

Definition 2.10. ([15]) Let (U, τ, D) be a soft space.

1. A family $C = \{(F_D)_i \mid i \in I\}$ of soft open sets in (U, τ, D) which satisfies $\underset{i\in I}{\cup}(F_D)_i = \widetilde{U}$ is called soft open cover of U. If a finite subfamily of a soft open cover C of U is also a soft open cover of U then it is called a finite sub-cover of C.
2. U is referred to as a soft compact if all soft open covers of U have a finite sub-cover.

Definition 2.11. ([20]) A soft space (U, τ, D) is called soft Hausdorff space provided that for any two distinct soft points $P^{x_1}_{d_1}, P^{x_2}_{d_2} \in SP(U, D)$ there exist members of τ F_D and G_E such that $P^{x_1}_{d_1} \widetilde{\in} F_D$, $P^{x_2}_{d_2} \widetilde{\in} G_E$ and $F_D \widetilde{\cap} G_E = \Phi$.

Proposition 2.12. ([33]) *Let F_D be a soft closed set on U where (U, τ, D) is a soft Hausdorff space then F_D is a soft compact set on U.*

In the published manuscripts, it was exhibited different classes of soft Hausdorff spaces with respect to distinct points (not soft points) which do not satisfy the above proposition; see, [21, 22, 32].

Theorem 2.13. ([27]) *Let (U,τ,D) be a soft space. Let $G_D \in S(U,D)$ be a soft set and $P^x_d \in SP(U,D)$. Then $P^x_d \widetilde{\in} \overline{G_D}$ if and only if there exists a soft net $\{P^{x_\alpha}_{d_\alpha} | \alpha \in A\}$ in G_D i.e. $P^{x_\alpha}_{d_\alpha} \widetilde{\in} G_D$ for all $\alpha \in A$ such that $P^{x_\alpha}_{d_\alpha} \to P^x_d$.*

Theorem 2.14. ([27]) *Let (U,τ_1,D) and (V,τ_2,E) be two soft spaces. A soft mapping $(K,n) : S(U,D) \to S(V,E)$ has a soft closed graph if and only if whenever $P^{x_\alpha}_{d_\alpha} \longrightarrow P^x_d$ in U and $(K,n)(P^{x_\alpha}_{d_\alpha}) = P^{K(x_\alpha)}_{n(d_\alpha)} \longrightarrow P^y_e$ in V then $P^y_e = (K,n)(P^x_d) = P^{K(x)}_{n(d)}$.*

Definition 2.15. ([37]) Let (U,τ_1,D) and (V,τ_2,E) be two soft spaces. Let $(K,n) : S(U,D) \to S(V,E)$ be a soft mapping and $P^x_d \in SP(U,D)$

1. (K,n) is a soft continuous at $P^x_d \in SP(U,D)$ if for each $G_E \in \mathcal{N}_\tau(P^{K(x)}_{n(d)})$ there exist $F_D \in \mathcal{N}_\tau(P^x_d)$ such that $(K,n)(F_D) \widetilde{\subseteq} G_E$.
2. (K,n) is a soft continuous if (K,n) is a soft continuous at each soft point in $SP(U,D)$.

Here, $\mathcal{N}_\tau(P^x_d)$ represents the family of all neighborhoods of a soft point P^x_d.

Theorem 2.16. ([37]) *Let (U,τ_1,D) and (V,τ_2,E) be two soft spaces. A soft mapping $(K,n) : S(U,D) \to S(V,E)$ is a soft continuous mapping if and only if $(K,n)^{-1}(F_E)$ is a soft closed set in U for all soft closed subsets F_E of V.*

3. Main results

We show the relationship between soft continuous mappings and soft graphs. Also, we introduce the definition of a soft filter generated by the soft net and vice-versa. We prove the equivalency of the convergence theory of soft filter and soft net. Therefore, results proved in [33, 35] for soft filter will also hold for soft nets.

First, we elucidate, by example below, that a soft continuous mapping may not have a soft closed graph.

Example 3.1. *Let $U = \{a,b,c\}$, $D = \{0,1\}$ and $\tau = \{\Phi, \widetilde{U}, (F_1,D), (F_2,D),$ $(F_3,D), (F_4,D), (F_5,D), (F_6,D)\}$ where,*

$F_1(0) = \{a\}, F_1(1) = \{c\};$

$F_2(0) = \{b\}, F_2(1) = \{a\};$

$F_3(0) = \{c\}, F_3(1) = \{b\};$

$F_4(0) = \{a,b\}, F_4(1) = \{a,c\};$

$F_5(0) = \{a,c\}, F_5(1) = \{b,c\};$

$F_6(0) = \{b,c\}, F_6(1) = \{a,b\}.$

Let $(K,n) : S(U,D) \to S(V,E)$ be soft mapping where $K = 1_U$ and $n = 1_D$. Then (K,n) is a soft continuous mapping but soft graph of (K,n), $\mathbb{G}(K,n) : D \times D \to \mathbf{P}(U \times U)$ defined by,

$$\mathbb{G}(K,n)(d,e) = \begin{cases} \Delta(K), & \text{if } d = e \\ \emptyset, & \text{if } d \neq e \end{cases}$$

is not soft closed in $U \times U$.

As in the characterization of soft closure in Theorem 2.13, the use of soft points below enables us to give a characterization of soft continuity which is not possible by using points of the soft set as shown in [25].

Theorem 3.2. *Let (U, τ_1, D) and (V, τ_2, E) be two soft spaces. A soft mapping $(K, n) : S(U, D) \to S(V, E)$ is a soft continuous mapping at $P_d^x \in SP(U, D)$ if and only if $P_{d_\alpha}^{x_\alpha} \longrightarrow P_d^x$ implies $P_{n(d_\alpha)}^{K(x_\alpha)} \longrightarrow P_{n(d)}^{K(x)}$.*

Proof. Let (K, n) be a soft continuous mapping at P_d^x and $G_E \in \mathcal{N}_\tau(P_{n(d)}^{K(x)})$. Then, we find a soft neighbourhood F_D of P_d^x such that $(K, n)(F_D) \widetilde{\subseteq} G_E$. If $P_{d_\alpha}^{x_\alpha} \longrightarrow P_d^x$, then soft net $\{P_{d_\alpha}^{x_\alpha} \mid \alpha \in A\}$ is eventually in F_D so $\{P_{n(d_\alpha)}^{K(x_\alpha)} \mid \alpha \in A\}$ is eventually in G_E. Therefore, $P_{n(d_\alpha)}^{K(x_\alpha)} \longrightarrow P_{n(d)}^{K(x)}$.

Conversely, suppose (K, n) is not a soft continuous at P_d^x, then there is a soft neighbourhood G_E of $P_{n(d)}^{K(x)}$ such that $(K, n)(H_D) \widetilde{\nsubseteq} G_E$ for every $H_D \in \mathcal{N}_\tau(P_d^x)$. Therefore, for every H_D there is some d_{H_D} such that $(K, n)(H_D)(d_{H_D}) \nsubseteq G_E(d_{H_D})$. Define a soft net, $T : \mathcal{N}_\tau(P_d^x) \to SP(U, D)$ as $T_{H_D} = P_{d_{H_D}}^{x_{H_D}}$ where $P_{d_{H_D}}^{x_{H_D}} \widetilde{\in} H_D$ for which $P_{n(d_{H_D})}^{K(x_{H_D})} \widetilde{\notin} G_E$. Now, $P_{d_{H_D}}^{x_{H_D}} \longrightarrow P_d^x$ but $P_{n(d_{H_D})}^{K(x_{H_D})}$ does not converge to $P_{n(d)}^{K(x)}$, which contradicts our assumption. □

For our next findings, we make use of the following.

Lemma 3.3. ([20]) *A soft space (U, τ, D) is a soft Hausdorff space iff any soft net in (U, τ, D) converges to one soft point at most.*

The following theorem demonstrates that if the co-domain of a soft continuous mapping is a soft Hausdorff, then that soft mapping has a soft closed graph.

Theorem 3.4. *Let $(K, n) : S(U, D) \to S(V, E)$ be a soft continuous mapping where (V, τ_2, E) is a soft Hausdorff space then $G(K, n)_{D\times E}$ is a soft closed set.*

Proof. Suppose $P_{(d,e)}^{(x,y)} \widetilde{\in} \overline{G(K, n)}_{D\times E}$, then by Theorem 2.13, there exist a soft net $P_{(d_\alpha,n(d_\alpha))}^{(x_\alpha,K(x_\alpha))}$ in $G(K, n)_{D\times E}$ such that $P_{(d_\alpha,n(d_\alpha))}^{(x_\alpha,K(x_\alpha))} \longrightarrow P_{(d,e)}^{(x,y)}$, which implies $P_{d_\alpha}^{x_\alpha} \longrightarrow P_d^x$ and $P_{n(d_\alpha)}^{K(x_\alpha)} \longrightarrow P_e^y$. Since (K, n) be a soft continuous, then by Theorem 3.2, $P_{n(d_\alpha)}^{K(x_\alpha)} \longrightarrow P_{n(d)}^{K(x)}$. Now, as $P_{n(d_\alpha)}^{K(x_\alpha)} \longrightarrow P_e^y$ and (V, τ_2, E) be a soft Hausdorff space then by Lemma 3.3, $P_e^y = P_{n(d)}^{K(x)}$ which implies $e = n(d)$ and $y = K(x)$ and therefore, $P_{(d,e)}^{(x,y)} \widetilde{\in} \mathbb{G}(K, n)_{D\times E}$. Hence, $G(K, n)_{D\times E}$ is a soft closed set. □

We shall see below (Corollary 3.7) that the converse of the above-mentioned theorem is also true. The following example shows that a soft mapping with soft closed graph need not be soft continuous if the domain is not soft compact.

Example 3.5. Let U be the set of real numbers $\mathbb{R}$. Let $D = \{d\}$ and $\tau = \{\Phi, \widetilde{U}\} \cup \{(F_O)_D\}$ where, $(F_O)_D = \{d, O\}$, where O is the usual open set in $\mathbb{R}$. Then (U, τ, D) is a soft space. Define $K : \mathbb{R} \to \mathbb{R}$ by, $K(x) = \begin{cases} 1/x, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ and $e : D \to D$ by $n(d) = d$. Then $(K, n) : S(U, D) \to S(U, D)$ be soft mapping and soft graph of (K, n), $G(K, n) : D\times D \to \mathbf{P}(U\times U)$ defined by, $G(K, n)(d, d) = \{(x, 1/x) | x \in \mathbb{R}\} \cup \{0, 0\}$, is a soft closed in $U \times U$. But (K, n) is not a soft continuous at P_d^0.

Wereason by the following theorem that the *soft equalizer* E_D of soft continuous mapping and a soft mapping with soft closed graph is a soft closed set.

Theorem 3.6. *Let $(K, n), (f, n') : S(U, D) \to S(V, E)$ be two soft mappings such that one of them has soft closed graph and the other is a soft continuous mapping and $S = \{P_d^x \in SP(U, D) \mid (K, n)(P_d^x) = (f, n')(P_d^x)\}$ be a subset of $SP(U, D)$ then the soft set $E_D = \widetilde{\bigcup}_{P_d^x \in S} P_d^x$ is a soft closed set in U.*

Proof. Assume (K, n) is a soft continuous mapping and (f, n') has soft closed graph. Let $P_d^x \widetilde{\in} \overline{E_D}$ then by Theorem 2.13, there exist a soft net $P_{d_\alpha}^{x_\alpha}$ in E_D such that $P_{d_\alpha}^{x_\alpha} \longrightarrow P_d^x$. Since (K, n) is a soft continuous,

$(K,n)(P^{x_\alpha}_{d_\alpha}) \longrightarrow (K,n)(P^x_d)$ by Theorem 3.2. Also, $P^{x_\alpha}_{d_\alpha} \widetilde{\in} F_D$ which implies $(K,n)(P^{x_\alpha}_{d_\alpha}) = (f,n')(P^{x_\alpha}_{d_\alpha})$. Now, (f,n') has a soft closed graph then $P^{x_\alpha}_{d_\alpha} \longrightarrow P^x_n$ and $(f,n')(P^{x_\alpha}_{d_\alpha}) \longrightarrow (K,n)(P^x_d)$ implies $(K,n)(P^x_d) = (f,n')(P^x_d)$. Therefore, $P^x_d \widetilde{\in} E_D$ and hence, E_D is a soft closed set in U. □

From Theorems 3.6 and 3.4 above we get the next.

Corollary 3.7. *Let $(K,n),(f,n') : S(U,D) \to S(V,E)$ be two soft continuous mappings and (V,τ_2,E) be a soft Hausdorff space. Then F_D defined in above theorem is a soft closed set in U.*

Definition 3.8. ([33]) A soft filter on U is a non-empty subset $\mathcal{L} \subseteq S(U,D)$ such that

1. $\Phi \notin \mathcal{L}$,
2. If F_D, $G_V \in \mathcal{L}$, then $F_D \widetilde{\cap} G_E \in \mathcal{L}$,
3. If $F_D \in \mathcal{L}$ and $F_D \widetilde{\subseteq} G_E$, then $G_E \in \mathcal{L}$.

Definition 3.9. ([33]) Let $\mathcal{L}_1$ and $\mathcal{L}_2$ be two soft filters on U. Then, $\mathcal{L}_2$ is finer than $\mathcal{L}_1$ (or $\mathcal{L}_1$ is coarser than $\mathcal{L}_2$) if $\mathcal{L}_1 \subseteq \mathcal{L}_2$.

Definition 3.10. ([33]) Let $\mathcal{L}$ be a soft filter on U. Then a subfamily C of $\mathcal{L}$ is called a soft filter base for $\mathcal{L}$ if for any $F_D \in \mathcal{L}$ there exist $G_E \in C$ such that $G_E \widetilde{\subseteq} F_D$.

Definition 3.11. ([20]) A soft filter $\mathcal{L}$ on a soft space (U,τ,D) is said to converge to $P^x_d \in SP(U,D)$, and we write $\mathcal{L} \longrightarrow P^x_d$, if $\mathcal{N}_\tau(P^x_d)\} \subseteq \mathcal{L}$.

For the application of results on soft filters, we define the following.

Definition 3.12. Let $\{P^{x_\alpha}_{d_\alpha} \mid \alpha \in A\}$ be a soft net in U and $\mathbf{B} = \{(G_{\alpha_0})_D = \widetilde{\bigcup}_{\alpha \gtrsim \alpha_0} \{P^{x_\alpha}_{d_\alpha} \mid \alpha_0 \in A\}$. Now, as for some α', $\alpha'' \in A$ there is some $\alpha^0 \gtrsim \alpha'$ and $\alpha^0 \gtrsim \alpha''$ such that $(G_{\alpha^0})_D \widetilde{\subseteq} (G_{\alpha'})_D \widetilde{\cap} (G_{\alpha''})_D$, $\mathbf{B}$ is a soft filter base and soft filter it generates is the associated soft filter of $\{P^{x_\alpha}_{d_\alpha} \mid \alpha \in D\}$.

Definition 3.13. Let $\mathcal{L}$ be a soft filter on U and let $D_{\mathcal{L}} = \{(P^x_d, F_D) \mid P^x_d \widetilde{\in} F_D \in \mathcal{L}\}$ then $(D_{\mathcal{L}}, \gtrsim)$ be a directed set where the relation $\gtrsim$ is defined by $(P^x_d, F_D) \gtrsim (P^x_d, G_D)$ if and only if $F_D \widetilde{\subseteq} G_D$. Then, the mapping $T : D_{\mathcal{L}} \to SP(U,D)$ defined by $T(P^x_d, F_D) = P^x_d$ is soft net generated by soft filter $\mathcal{L}$.

We have established how to generate a soft net from the soft filter and a soft filter from the soft net. In order to get some results, we must also show their convergence results correspond to each other.

Proposition 3.14. *Let (U,τ,D) be a soft space.*

1. *A soft net $P^{x_\alpha}_{d_\alpha} \longrightarrow P^x_d$ if and only if its associated soft filter $\mathcal{L} \longrightarrow P^x_d$.*
2. *A soft filter $\mathcal{L} \longrightarrow P^x_d$ if and only if its associated soft net $P^{x_\alpha}_{d_\alpha} \longrightarrow P^x_d$.*

Proof. 1. Let $\{P^{x_\alpha}_{d_\alpha} \mid \alpha \in A\}$ be a soft net such that $P^{x_\alpha}_{d_\alpha} \longrightarrow P^x_d$ and $F_D \in \mathcal{N}_\tau(P^x_d)$ then there exist $\alpha_0 \in A$ such that $P^{x_\alpha}_{d_\alpha} \in F_D$ for all $\alpha \gtrsim \alpha_0$ which implies $\widetilde{\bigcup}_{\alpha \gtrsim \alpha_0} P^{x_\alpha}_{d_\alpha} \widetilde{\subseteq} F_D$. Now $(G_{\alpha_0})_D = \widetilde{\bigcup}_{\alpha \gtrsim \alpha_0} P^{x_\alpha}_{d_\alpha} \in \mathbf{B}$, where $\mathbf{B}$ is a soft filterbase for $\mathcal{L}$. Hence $F_D \in \mathcal{L}$ and therefore, $\mathcal{L} \longrightarrow P^x_d$.

Conversely, assume F_D be soft neighbourhood of P^x_d then $F_D \in \mathcal{L}$ then there exist $G_D \in \mathbf{B}$ such that $G_D \widetilde{\subseteq} F_D$ where $G_D = (G_{\alpha_0})_D = \widetilde{\bigcup}_{\alpha \gtrsim \alpha_0} P^{x_\alpha}_{d_\alpha} \widetilde{\subseteq} F_D$ which implies $P^{x_\alpha}_{d_\alpha} \widetilde{\in} F_D$ for every $\alpha \gtrsim \alpha_0$. Hence $P^{x_\alpha}_{d_\alpha} \longrightarrow P^x_d$.

2. Let $\mathcal{L} \longrightarrow P^x_d$ and F_D be soft neighbourhood of P^x_d. Then $F_D \in \mathcal{L}$ implies $(P^x_d, F_D) \in D_{\mathcal{L}}$ where $T : D_{\mathcal{L}} \to SP(U,D)$ be a soft net then for $(P^y_e, G_D) \gtrsim (P^x_d, F_D)$, $T(P^y_e, G_E) = P^y_e \widetilde{\in} G_D \widetilde{\subseteq} F_D$ implies $T(P^y_e, G_D) \widetilde{\in} F_D$.

Conversely, let $T \to P^x_d$ and F_D be soft neighbourhood of P^x_d then there exist $(P^{y_0}_{e_0}, G_D) \in D_{\mathcal{L}}$ where $G_D \in \mathcal{L}$ such that for every $(P^y_e, H_D) \gtrsim (P^{y_0}_{e_0}, G_D)$, $T(P^y_e, H_D) \widetilde{\in} F_D$. In particular, for every $P^{y'}_{e'} \widetilde{\in} G_D$ and $(P^{y'}_{e'}, G_D) \gtrsim (P^{y_0}_{e_0}, G_D)$. We get $T(P^{y'}_{e'}, G_D) \widetilde{\in} F_D$ implies $P^{y'}_{e'} \widetilde{\in} F_D$ and which further implies $G_D \widetilde{\subseteq} F_D$. Hence, $F_D \in \mathcal{L}$. □

Definition 3.15. A soft point P_d^x is termed a soft set point of soft net $\{P_{d_\alpha}^{x_\alpha} \mid \alpha \in A\}$ if for every $F_D \in \mathcal{N}_\tau(P_d^x)$ and $\alpha_0 \in A$ there exists $\alpha \gtrsim \alpha_0$ such that $P_{d_\alpha}^{x_\alpha} \widetilde{\in} F_D$.

Definition 3.16. ([35]) Let $\mathcal{L}$ be a soft filter on a soft space (U, τ, D). A soft point P_d^x is named a soft cluster point of $\mathcal{L}$ if $P_d^x \widetilde{\in} \overline{G_D}$ for every $G_D \in \mathcal{L}$.

Proposition 3.17. *Let (U, τ, D) is a soft space.*

1. P_d^x *is a soft set point of soft net* $\{P_{d_\alpha}^{x_\alpha} \mid \alpha \in A\}$ *if and only if* P_d^x *is a soft set point of the associated soft filter* $\mathcal{L}$.
2. P_d^x *is a soft set point of soft filter* $\mathcal{L}$ *if and only if* P_d^x *is a soft cluster point of the associated soft net* $\{P_{d_\alpha}^{x_\alpha} \mid \alpha \in A\}$.

Definition 3.18. Let U be a set and $(X, \gtrsim)$ and $(Y, \gtrsim)$ be two directed sets. Let $T : X \to SP(U, D)$ be a soft net and $\eta : Y \to D$ be a map. Suppose for every $\alpha_0 \in A$ there exist $\mu_0 \in Y$ such that $\eta(\mu) \gtrsim \alpha_0$ whenever $\mu \gtrsim \mu_0$, then $To\eta : Y \to SP(U, D)$ be a soft subnet of the soft net T. We write $\eta(\mu) = \alpha_\mu$ and $(To\eta)(\mu) = T(\eta(\mu)) = P_{d_{\alpha_\mu}}^{x_{\alpha_\mu}}$.

Lemma 3.19. ([35]) *For a soft space (U, τ, D), the following are equivalent:*

1. *A soft space (U, τ, D) is a soft compact.*
2. *Every soft filter on U has a soft set point.*
3. *Every maximal soft filter on U converges to a soft point.*

Theorem 3.20. *Let $(K, n) : S(U, D) \to S(V, E)$ be a soft mapping and (K, n) has a soft closed graph then $(K, n)^{-1}(F_D)$ is soft closed set in U for all soft compact subsets F_E of V.*

Proof. Let F_E be a soft compact subset of U and $(K, n)^{-1}(F_E)$ is not a soft closed in U. Therefore, there exist $P_d^x \widetilde{\in} \overline{(K, n)^{-1}(F_E)}$ such that $P_d^x \widetilde{\notin} (K, n)^{-1}(F_E)$ then by Theorem 3.2, there exist a soft net $P_{d_\alpha}^{x_\alpha}$ in $(K, n)^{-1}(F_E)$ such that $P_{d_\alpha}^{x_\alpha} \longrightarrow P_d^x$. This implies $(K, n)(P_{d_\alpha}^{x_\alpha}) = P_{n(d_\alpha)}^{K(x_\alpha)}$ is a soft net in (F_E). Now, (F_E) is a soft compact then by Lemma 3.19, $(K, n)(P_{d_\alpha}^{x_\alpha})$ has a soft subnet $(K, n)(P_{d_{\alpha_\mu}}^{x_{\alpha_\mu}})$ which converges to $P_e^y \widetilde{\in} F_E$. This implies $P_{d_{\alpha_\mu}}^{x_{\alpha_\mu}}$ is a soft subnet of $P_{d_\alpha}^{x_\alpha}$ and therefore, $P_{d_{\alpha_\mu}}^{x_{\alpha_\mu}} \longrightarrow P_d^x$. Since, $(K, n)(P_{d_{\alpha_\mu}}^{x_{\alpha_\mu}}) \longrightarrow P_e^y$ and (K, n) has a soft closed graph, $P_e^y = (K, n)(P_d^x)$ by Theorem 2.14. This implies $P_d^x \widetilde{\in} (K, n)^{-1}(P_e^y) \widetilde{\subseteq} (K, n)^{-1}(F_E)$ which contradicts our supposition. □

Finally from Theorem 2.16 and Theorem 3.20 above, we have the following converse of Theorem 3.4.

Corollary 3.21. *Let $(K, n) : S(U, D) \to S(V, E)$ be a soft mapping with soft closed graph and (V, τ_2, E) is a soft compact space then (K, n) is a soft continuous.*

4. Conclusion

In this study, we have presented the properties and characterizations of continuous mappings in terms of soft closed graphs. The central focus of this paper revolves around the investigation of soft mappings with soft closed graphs, particularly in the context of soft compact spaces. We have proved a significant result that emphasizes the connection between the continuity of soft mappings and soft closed graphs. This result establishes a connection between the topological properties of soft compact spaces and the continuity of soft mappings with closed graphs. Our findings not only advance the understanding of soft continuous mappings but also have broader implications for the development of soft set theory. The work creates chances to conduct further study on the relationships between different mathematical structures and soft topology. On the other hand, the vital notions of topology such as soft closed graphs and soft set points are used in various applications including image processing and pattern recognition. It can also provide applications in computer algorithms including modeling and simulation.

To make a complete frame for the ideas presented herein, one can examine the behaviors of soft closed graphs when soft Hausdorff space is defined in terms of ordinary points and partial belonging relation as follows (U, τ, D) is called a soft Hausdorff space if for any two distinct ordinary points $x_1, x_2 \in SP(U, D)$ there exist disjoint soft open sets F_D and G_E such that

$$x_1 \Subset F_D,\ x_2 \Subset G_E, \text{and}$$

$$x_1 \notin F_D,\ x_2 \notin G_E.$$

Of course, it will be obtained new characterizations since some classical properties of Hausdorff spaces are evaporated in such kinds of soft Hausdorff spaces.

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