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On a solvable difference equations system

Ömer Aktaş[a,*], Merve Kara[a], Yasin Yazlik[b]

[a]*Department of Mathematics, Kamil Ozdag Science Faculty, Karamanoğlu Mehmetbey University, 70200 Karaman, Turkey*
[b]*Department of Mathematics, Faculty of Arts and Sciences, Nevşehir Hacı Bektaş Veli University, 50300, Nevşehir, Turkey*

Abstract. In this paper, we study three dimensional system of difference equations. Firstly, we examine the solutions of the mentioned system depending on whether the parameters are equal to zero or non-zero. In addition, the solutions of this system are obtained in closed form. Finally, we also describe the forbidden set of the solutions of the system of difference equations.

1. Introduction and preliminaries

First of all, recall that $\mathbb{N}$, $\mathbb{N}_0$, $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{C}$, stand for the set of natural, non-negative integer, integer, real and complex numbers, respectively. If $k, l \in \mathbb{Z}$, $k \leq l$, the notation $i = \overline{k, l}$ stands for $\{i \in \mathbb{Z} : k \leq i \leq l\}$.

In the recent years, the interest of difference equations and their systems has increased. Researchers are interested in solutions and behavior of solutions of difference equations and their systems [3, 4, 6, 10–13, 15, 22, 23, 25, 27–30, 32, 33]. One of difference equations is homogeneous linear difference equations with constant coefficients, which is in the following form:

$$s_n = a s_{n-3}, \quad n \in \mathbb{N}_0, \tag{1}$$

where the initial values s_{-3}, s_{-2}, s_{-1} and the parameter a are real numbers. The solution of linear difference equation (1) is given

$$s_{3m+i} = a^{m+1} s_{i-3}, \quad m \in \mathbb{N}_0, \tag{2}$$

for $i \in \{0, 1, 2\}$.
Another difference equation is non-homogeneous linear difference equation with constant coefficients, which is in the following form:

$$s_n = a s_{n-3} + b, \quad n \in \mathbb{N}_0, \tag{3}$$

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This study is a part of the first author's Ph.D. Thesis.
* Corresponding author: Ömer Aktaş
Email addresses: aktas.omer10@gmail.com (Ömer Aktaş), mervekara@kmu.edu.tr (Merve Kara), yyazlik@nevsehir.edu.tr (Yasin Yazlik)
ORCID iDs: https://orcid.org/0000-0002-5763-0308 (Ömer Aktaş), https://orcid.org/0000-0001-8081-0254 (Merve Kara), https://orcid.org/0000-0001-6369-540X (Yasin Yazlik)

where the initial values s_{-3}, s_{-2}, s_{-1} and the parameters a, b are real numbers. The solution of equation (3) can be written as follows

$$s_{3m+i} = \begin{cases} \frac{b+a^{m+1}((1-a)s_{i-3}-b)}{1-a}, & a \neq 1, \\ s_{i-3} + b(m+1), & a = 1, \end{cases} \tag{4}$$

where $m \in \mathbb{N}_0$ and $i \in \{0, 1, 2\}$.
Kara and Yazlik in [21], obtained the solutions of the following difference equations

$$x_n = \frac{x_{n-k}x_{n-k-l}}{x_{n-l}(a_n + b_n x_{n-k}x_{n-k-l})}, \quad n \in \mathbb{N}_0, \tag{5}$$

where the initial values are real numbers. Also they studied the forbidden set and the asymptotic behavior of equation (5). Moreover, the case $k = 3$, $l = 1$, $a_n = \pm 1 = b_n$, in equation (5), which is studied by Alzubaidi and Elsayed in [2]. In addition, the case $k = 3$, $l = 4$, $a_n = \pm 1 = b_n$, in equation (5), which is studied by Bukhari and Elsayed in [5].
Yazlik and Güngör in [34] obtained the solution of the following difference equation

$$x_n = \frac{x_{n-4}x_{n-5}x_{n-6}}{x_{n-1}x_{n-2}(a + bx_{n-3}x_{n-4}x_{n-5}x_{n-6})}, \quad n \in \mathbb{N}_0, \tag{6}$$

where the initial values and the parameters are real numbers. Also, they investigated the asymptotic behavior of the solution of equation (6).
Some authors studied two dimensional system of difference equations. For instance, Touafek and Elsayed investigated the solutions of the following systems of difference equations

$$x_{n+1} = \frac{y_n}{x_{n-1}(\pm 1 \pm y_n)}, \quad y_{n+1} = \frac{x_n}{x_{n-1}(\pm 1 \pm x_n)}, \quad n \in \mathbb{N}_0,$$

where the initial values are nonzero real numbers in [31].
Kara and Yazlik in [19], investigated the solutions of the following system of difference equations

$$x_n = \frac{x_{n-k}y_{n-k-l}}{y_{n-l}(a_n + b_n x_{n-k}y_{n-k-l})}, \ y_n = \frac{y_{n-k}x_{n-k-l}}{x_{n-l}(\alpha_n + \beta_n y_{n-k}x_{n-k-l})}, \ n \in \mathbb{N}_0, \tag{7}$$

where the initial values are real numbers and $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$, $(\alpha_n)_{n\in\mathbb{N}_0}$, $(\beta_n)_{n\in\mathbb{N}_0}$ are the sequences of real numbers. Moreover, the asymptotic behavior of well-defined solutions of system (7) was studied for the case $k = 2$, $l = k$.
There are the particular cases of system (7) in literature. For example, the case $k = 2$, $l = 2$, $a_n = -1 = b_n$, $\alpha_n = \pm 1 = \beta_n$, in equation (7), which is investigated by Almatrafi in [1]. Similarly, the authors of [18] solved the case $k = 2$, $l = 1$, in equation (7). Furthermore, they described the asymptotic behavior and the periodicity of the solutions when all sequences are constant. There are some systems of difference equations which are not particular cases of system (7). Such as Halim et. al. investigated the solutions of the following system of difference equations

$$x_{n+1} = \frac{y_{n-1}x_{n-2}}{y_n(a + by_{n-1}x_{n-2})}, \quad y_{n+1} = \frac{x_{n-1}y_{n-2}}{x_n(a + bx_{n-1}y_{n-2})}, \quad n \in \mathbb{N}_0,$$

where the initial values and the parameters are real numbers in [14].
The authors of [9] solved the following systems of difference equations

$$x_{n+1} = \frac{y_{n-5}x_{n-8}}{y_{n-2}(1 + y_{n-5}x_{n-8})}, \quad y_{n+1} = \frac{x_{n-5}y_{n-8}}{x_{n-2}(\pm 1 \pm x_{n-5}y_{n-8})}, \quad n \in \mathbb{N}_0,$$

where the initial values are nonzero real numbers.
Karakaya et. al. obtained the solutions of the following system of difference equations

$$x_n = \frac{x_{n-4}y_{n-5}x_{n-6}}{y_{n-1}x_{n-2}(a + by_{n-3}x_{n-4}y_{n-5}x_{n-6})}, \quad y_n = \frac{y_{n-4}x_{n-5}y_{n-6}}{x_{n-1}y_{n-2}(c + dx_{n-3}y_{n-4}x_{n-5}y_{n-6})}, \quad n \in \mathbb{N}_0, \tag{8}$$

where the initial values and the parameters are real numbers. Also, they defined the forbidden set of the solutions of system (8) in [26].
A lot of mathematicians solved three-dimensional systems of difference equations with constant or variable coefficients. For example, Elsayed et.al. in [8], gained the solutions of the following systems of the difference equations

$$P_{n+1}=\frac{P_{n-4}S_{n-2}Q_n}{S_{n-3}Q_{n-1}\left(1\pm P_{n-4}S_{n-2}Q_n\right)},Q_{n+1}=\frac{Q_{n-4}P_{n-2}S_n}{P_{n-3}S_{n-1}\left(1\pm Q_{n-4}P_{n-2}S_n\right)},S_{n+1}=\frac{S_{n-4}Q_{n-2}P_n}{Q_{n-3}P_{n-1}\left(1\pm S_{n-4}Q_{n-2}P_n\right)},$$

for $n\in\mathbb{N}_0$, the initial values are nonzero real numbers.
In addition, Kara and Aktaş in [17], investigated the solutions of the following system of difference equations

$$x_n=\frac{x_{n-2}y_{n-3}}{y_{n-1}\left(a+bx_{n-2}y_{n-3}\right)},y_n=\frac{y_{n-2}z_{n-3}}{z_{n-1}\left(c+dy_{n-2}z_{n-3}\right)},z_n=\frac{z_{n-2}x_{n-3}}{x_{n-1}\left(e+fz_{n-2}x_{n-3}\right)},\quad n\in\mathbb{N}_0, \tag{9}$$

where the parameters and the initial values are nonzero real numbers.
The following system of difference equations with variable coefficients

$$x_n=\frac{x_{n-2}z_{n-3}}{z_{n-1}\left(a_n+b_nx_{n-2}z_{n-3}\right)},y_n=\frac{y_{n-2}x_{n-3}}{x_{n-1}\left(\alpha_n+\beta_ny_{n-2}x_{n-3}\right)},z_n=\frac{z_{n-2}y_{n-3}}{y_{n-1}\left(A_n+B_nz_{n-2}y_{n-3}\right)},\quad n\in\mathbb{N}_0,$$

was studied by Kara and Yazlik in [20], where the initial values are nonzero real numbers and $(a_n)_{\in\mathbb{N}_0}$, $(b_n)_{\in\mathbb{N}_0}$, $(\alpha_n)_{\in\mathbb{N}_0}$, $(\beta_n)_{\in\mathbb{N}_0}$, $(A_n)_{\in\mathbb{N}_0}$, $(B_n)_{\in\mathbb{N}_0}$, are nonzero real numbers sequences. Also, they defined the forbidden set of the solutions and obtained the periodic solutions of the aforementioned system.
Recently, Kara obtained the solutions of the following system of difference equations

$$\begin{cases}x_n= & \frac{y_{n-4}z_{n-5}}{y_{n-1}\left(a_n+b_nz_{n-2}x_{n-3}y_{n-4}z_{n-5}\right)},\\ y_n= & \frac{z_{n-4}x_{n-5}}{z_{n-1}\left(\alpha_n+\beta_nx_{n-2}y_{n-3}z_{n-4}x_{n-5}\right)},\ n\in\mathbb{N}_0,\\ z_n= & \frac{x_{n-4}y_{n-5}}{x_{n-1}\left(A_n+B_ny_{n-2}z_{n-3}x_{n-4}y_{n-5}\right)},\end{cases} \tag{10}$$

where $(a_n)_{\in\mathbb{N}_0}$, $(b_n)_{\in\mathbb{N}_0}$, $(\alpha_n)_{\in\mathbb{N}_0}$, $(\beta_n)_{\in\mathbb{N}_0}$, $(A_n)_{\in\mathbb{N}_0}$, $(B_n)_{\in\mathbb{N}_0}$ and the initial values are real numbers. Also, she investigated the forbidden set of the solutions of system (10) in [16].
Lately, Kara et.al. in [24], got the solutions of the following system of difference equations

$$\begin{cases}x_{n+1}= & \frac{\prod_{j=0}^{k}z_{n-3j}}{\prod_{j=1}^{k}x_{n-(3j-1)}\left(a_n+b_n\prod_{j=0}^{k}z_{n-3j}\right)},\\ y_{n+1}= & \frac{\prod_{j=0}^{k}x_{n-3j}}{\prod_{j=1}^{k}y_{n-(3j-1)}\left(c_n+d_n\prod_{j=0}^{k}x_{n-3j}\right)},\quad n\in\mathbb{N}_0,\\ z_{n+1}= & \frac{\prod_{j=0}^{k}y_{n-3j}}{\prod_{j=1}^{k}z_{n-(3j-1)}\left(e_n+f_n\prod_{j=0}^{k}y_{n-3j}\right)},\end{cases} \tag{11}$$

where the initial values and the sequences are real numbers. Furthermore, they described the forbidden set of the solutions of system (11).
Motivated by these papers, we study three dimensional form of system (8) as follows:

$$\begin{cases}x_n= & \frac{y_{n-4}z_{n-5}x_{n-6}}{y_{n-1}z_{n-2}\left(\alpha+\beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}\right)},\\ y_n= & \frac{z_{n-4}x_{n-5}y_{n-6}}{z_{n-1}x_{n-2}\left(\gamma+\theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}\right)},\ n\in\mathbb{N}_0,\\ z_n= & \frac{x_{n-4}y_{n-5}z_{n-6}}{x_{n-1}y_{n-2}\left(\eta+\zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}\right)},\end{cases} \tag{12}$$

where the initial values x_{-p}, y_{-p}, z_{-p} for $p=\overline{1,6}$ and the parameters α, β, γ, θ, η, ζ are real numbers. Note that system (12) is a natural extension of both equation (6) and system (8).

Definition 1.1. *[7] Let $(x_n, y_n, z_n)_{n\geq -6}$ be a solution of a system* (12)*. The solution $(x_n, y_n, z_n)_{n\geq -6}$ is called eventually periodic with period p if $x_{n+p}=x_n$, $y_{n+p}=y_n$ and $z_{n+p}=z_n$ for all $n\geq n_0$ where $n_0\in\mathbb{Z}$. If $n_0=-6$ is said that the solution is periodic with period p.*

2. The Solutions of the particular cases of system (12)

We will deal with well-defined solutions to system (12). Hence, we suppose that

$$x_n \neq 0, \quad y_n \neq 0, \quad z_n \neq 0, \quad n \geq -6,$$

and

$$\alpha + \beta x_{n-3} y_{n-4} z_{n-5} x_{n-6} \neq 0, \quad \gamma + \theta y_{n-3} z_{n-4} x_{n-5} y_{n-6} \neq 0, \quad \eta + \zeta z_{n-3} x_{n-4} y_{n-5} z_{n-6} \neq 0, \quad n \in \mathbb{N}_0.$$

It is obvious that if $x_{-p} = 0$ or $y_{-p} = 0$ or $z_{-p} = 0$, for some $p \in \{1, 2\}$, then x_0 or y_0 or z_0 is not defined. Likely, if $x_{-3} = 0$ (or $y_{-3} = 0$ (or $z_{-3} = 0$)), then $z_1 = 0$ (or $x_1 = 0$ (or $y_1 = 0$)) and y_2 (or z_2 (or x_2)) is not defined, respectively while if $x_{-q} = 0$, $y_{-q} = 0$, $z_{-q} = 0$, for some $q \in \{4, 5, 6\}$, then $x_0 = 0$, or $y_0 = 0$, $z_0 = 0$, so that x_1 or y_1 or z_1 is not defined. On the other hand, we assume that $x_{n_0} = 0$, for some $n_0 \in \mathbb{N}_0$, and $x_n \neq 0$, for every $n < n_0$, then from system (12) it follows that $y_{n_0-4} = 0$ or $z_{n_0-5} = 0$, that is contradiction. For this reason, the set

$$\left\{\vec{\mathbb{F}} : x_{-p} = 0 \quad \text{or} \quad y_{-p} = 0 \quad \text{or} \quad z_{-p} = 0, \quad p \in \{1, 2, 3, 4, 5, 6\}\right\}$$

is a subset of forbidden set of the solutions of the initial values of system (12). Thus, for well-defined solutions of system (12), $(x_n, y_n, z_n)_{n \geq -6}$, we get

$$x_n y_n z_n \neq 0, \quad n \geq -6,$$

if and only if $x_{-i} y_{-i} z_{-i} \neq 0$, $i \in \{1, 2, 3, 4, 5, 6\}$.

There are 64 cases depending on whether the parameters are equal to zero or not. In 37 cases out of the 64 mentioned cases, system (12) is not defined. For example, Case $\alpha = \beta = 0$, $\gamma\theta\eta\zeta \neq 0$, Case $\alpha = \beta = \gamma = 0$, $\theta\eta\zeta \neq 0$, Case $\alpha = \beta = \gamma = \theta = 0$, $\eta\zeta \neq 0$, Case $\alpha = \beta = \gamma = \theta = \eta = 0$, $\zeta \neq 0$, Case $\alpha = \beta = \gamma = \theta = \eta = \zeta = 0$, etc.

In the rest of this study, we show solvability of the following 27 systems of difference equations by presenting closed-form formulas for their well-defined solutions.

Case 1: Let $\alpha = \gamma = \eta = 0$ and $\beta\theta\zeta \neq 0$. In this case, system (12) transforms into the following system

$$x_n = \frac{1}{\beta y_{n-1} z_{n-2} x_{n-3}}, \quad y_n = \frac{1}{\theta z_{n-1} x_{n-2} y_{n-3}}, \quad z_n = \frac{1}{\zeta x_{n-1} y_{n-2} z_{n-3}}, \quad n \in \mathbb{N}_0. \tag{13}$$

Multiplying the first equation in system (13) by y_{n-1} for all $n \in \mathbb{N}_0$, the second by z_{n-1} for all $n \in \mathbb{N}_0$ and the third by x_{n-1} for all $n \in \mathbb{N}_0$, it follows that

$$x_n y_{n-1} = \frac{1}{\beta z_{n-2} x_{n-3}}, \quad y_n z_{n-1} = \frac{1}{\theta x_{n-2} y_{n-3}}, \quad z_n x_{n-1} = \frac{1}{\zeta y_{n-2} z_{n-3}}, \quad n \in \mathbb{N}_0. \tag{14}$$

Using the change of variables

$$x_n y_{n-1} = u_n, \quad y_n z_{n-1} = \hat{u}_n, \quad z_n x_{n-1} = \tilde{u}_n, \quad n \geq -2, \tag{15}$$

system (14) becomes

$$u_n = \frac{1}{\beta \tilde{u}_{n-2}}, \quad \hat{u}_n = \frac{1}{\theta u_{n-2}}, \quad \tilde{u}_n = \frac{1}{\zeta \hat{u}_{n-2}}, \quad n \in \mathbb{N}_0. \tag{16}$$

From (16), we have

$$u_n = u_{n-12}, \quad \hat{u}_n = \hat{u}_{n-12}, \quad \tilde{u}_n = \tilde{u}_{n-12}, \quad n \geq 10,$$

which means that $(u_n)_{n\geq -2}$, $(\hat{u}_n)_{n\geq -2}$ and $(\tilde{u}_n)_{n\geq -2}$, are twelve-periodic, that is,

$$u_{12m+j} = u_{j-12}, \quad \hat{u}_{12m+j} = \hat{u}_{j-12}, \quad \tilde{u}_{12m+j} = \tilde{u}_{j-12}, \quad m \in \mathbb{N}_0,$$

where $j = \overline{-2, 9}$.
From (15), we attain

$$x_{12n+i} = x_{12(n-1)+i}, \quad y_{12n+i} = y_{12(n-1)+i}, \quad z_{12n+i} = z_{12(n-1)+i}, \quad n \in \mathbb{N}_0,$$

where $i = \overline{9, 20}$, and we get

$$x_{12m+i} = x_{i-12}, \quad y_{12m+i} = y_{i-12}, \quad z_{12m+i} = z_{i-12}, \tag{17}$$

for every $m \in \mathbb{N}_0$ and $i = \overline{9, 20}$.

Case 2: Let $\beta = \theta = \zeta = 0$ and $\alpha\gamma\eta \neq 0$ In this case, system (12) is expressed as

$$x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{\alpha y_{n-1}z_{n-2}}, \quad y_n = \frac{z_{n-4}x_{n-5}y_{n-6}}{\gamma z_{n-1}x_{n-2}}, \quad z_n = \frac{x_{n-4}y_{n-5}z_{n-6}}{\eta x_{n-1}y_{n-2}}, \quad n \in \mathbb{N}_0. \tag{18}$$

Multiplying the first equation in system (18) by $y_{n-1}z_{n-2}x_{n-3}$, for all $n \in \mathbb{N}_0$, the second by $z_{n-1}x_{n-2}y_{n-3}$, for all $n \in \mathbb{N}_0$ and the third by $x_{n-1}y_{n-2}z_{n-3}$, for all $n \in \mathbb{N}_0$, it follows that

$$x_n y_{n-1}z_{n-2}x_{n-3} = \frac{x_{n-3}y_{n-4}z_{n-5}x_{n-6}}{\alpha}, y_n z_{n-1}x_{n-2}y_{n-3} = \frac{y_{n-3}z_{n-4}x_{n-5}y_{n-6}}{\gamma}, z_n x_{n-1}y_{n-2}z_{n-3} = \frac{z_{n-3}x_{n-4}y_{n-5}z_{n-6}}{\eta}, \tag{19}$$

for $n \in \mathbb{N}_0$.
By using the change of variables

$$x_n y_{n-1}z_{n-2}x_{n-3} = w_n, \quad y_n z_{n-1}x_{n-2}y_{n-3} = \hat{w}_n, \quad z_n x_{n-1}y_{n-2}z_{n-3} = \tilde{w}_n, \quad n \geq -3, \tag{20}$$

system (19) becomes

$$w_n = \frac{1}{\alpha}w_{n-3}, \quad \hat{w}_{n=}\frac{1}{\gamma}\hat{w}_{n-3}, \quad \tilde{w}_n = \frac{1}{\eta}\tilde{w}_{n-3}, \quad n \in \mathbb{N}_0. \tag{21}$$

From (2), the solutions of equations in (21) are the following form

$$w_{3m+i} = \left(\frac{1}{\alpha}\right)^{m+1} w_{i-3}, \quad \hat{w}_{3m+i} = \left(\frac{1}{\gamma}\right)^{m+1} \hat{w}_{i-3}, \quad \tilde{w}_{3m+i} = \left(\frac{1}{\eta}\right)^{m+1} \tilde{w}_{i-3}, \quad m \in \mathbb{N}_0, \tag{22}$$

for $i \in \{0, 1, 2\}$.
From (20), we get

$$x_n = \frac{w_n\hat{w}_{n-4}\tilde{w}_{n-8}}{\hat{w}_{n-1}\tilde{w}_{n-5}w_{n-9}}x_{n-12}, \quad y_n = \frac{\hat{w}_n\tilde{w}_{n-4}w_{n-8}}{\tilde{w}_{n-1}w_{n-5}\hat{w}_{n-9}}y_{n-12}, \quad z_n = \frac{\tilde{w}_n w_{n-4}\hat{w}_{n-8}}{w_{n-1}\hat{w}_{n-5}\tilde{w}_{n-9}}z_{n-12}, \quad n \geq 6, \tag{23}$$

and therefore

$$\begin{cases} x_{12m+j} = \frac{w_{12m+j}\hat{w}_{12m+j-4}\tilde{w}_{12m+j-8}}{\hat{w}_{12m+j-1}\tilde{w}_{12m+j-5}w_{12m+j-9}}x_{12(m-1)+j}, \\ y_{12m+j} = \frac{\hat{w}_{12m+j}\tilde{w}_{12m+j-4}w_{12m+j-8}}{\tilde{w}_{12m+j-1}w_{12m+j-5}\hat{w}_{12m+j-9}}y_{12(m-1)+j}, \quad m \in \mathbb{N}_0, \\ z_{12m+j} = \frac{\tilde{w}_{12m+j}w_{12m+j-4}\hat{w}_{12m+j-8}}{w_{12m+j-1}\hat{w}_{12m+j-5}\tilde{w}_{12m+j-9}}z_{12(m-1)+j}, \end{cases} \tag{24}$$

for $j = \overline{6, 17}$.
From (24), we attain

$$\begin{aligned} x_{12m+3r+s} &= x_{3r+s-12} \prod_{p=0}^{m} \frac{w_{12p+3r+s}\hat{w}_{12p+3r+s-4}\tilde{w}_{12p+3r+s-8}}{\hat{w}_{12p+3r+s-1}\tilde{w}_{12p+3r+s-5}w_{12p+3r+s-9}}, \\ y_{12m+3r+s} &= y_{3r+s-12} \prod_{p=0}^{m} \frac{\hat{w}_{12p+3r+s}\tilde{w}_{12p+3r+s-4}w_{12p+3r+s-8}}{\tilde{w}_{12p+3r+s-1}w_{12p+3r+s-5}\hat{w}_{12p+3r+s-9}}, \\ z_{12m+3r+s} &= z_{3r+s-12} \prod_{p=0}^{m} \frac{\tilde{w}_{12p+3r+s}w_{12p+3r+s-4}\hat{w}_{12p+3r+s-8}}{w_{12p+3r+s-1}\hat{w}_{12p+3r+s-5}\tilde{w}_{12p+3r+s-9}}, \end{aligned} \tag{25}$$

where $m \in \mathbb{N}_0$, $r \in \{2, 3, 4, 5\}$ and $s \in \{0, 1, 2\}$.
From (22) and (25), we obtain

$$x_{12m+3r+s} = \left(\frac{\gamma\eta}{\alpha^3}\right)^{m+1} x_{3r+s-12}, \quad y_{12m+3r+s} = \left(\frac{\alpha\eta}{\gamma^3}\right)^{m+1} y_{3r+s-12}, \quad z_{12m+3r+s} = \left(\frac{\alpha\gamma}{\eta^3}\right)^{m+1} z_{3r+s-12}, \tag{26}$$

where $m \in \mathbb{N}_0$, $r \in \{2, 3, 4, 5\}$ and $s \in \{0, 1, 2\}$.

Case 3: Let $\alpha = \gamma = \zeta = 0$ and $\beta\theta\eta \neq 0$. In this case, system (12) turns into the following system

$$x_n = \frac{1}{\beta y_{n-1}z_{n-2}x_{n-3}}, \quad y_n = \frac{1}{\theta z_{n-1}x_{n-2}y_{n-3}}, \quad z_n = \frac{x_{n-4}y_{n-5}z_{n-6}}{\eta x_{n-1}y_{n-2}}, \quad n \in \mathbb{N}_0. \tag{27}$$

Multiplying the first equation in system (27) by $y_{n-1}z_{n-2}x_{n-3}$ for all $n \in \mathbb{N}_0$, the second by $z_{n-1}x_{n-2}y_{n-3}$ for all $n \in \mathbb{N}_0$ and the third by $x_{n-1}y_{n-2}z_{n-3}$ for all $n \in \mathbb{N}_0$, it follows that

$$x_n y_{n-1}z_{n-2}x_{n-3} = \frac{1}{\beta}, \quad y_n z_{n-1}x_{n-2}y_{n-3} = \frac{1}{\theta}, \quad z_n x_{n-1}y_{n-2}z_{n-3} = \frac{z_{n-3}x_{n-4}y_{n-5}z_{n-6}}{\eta}, \quad n \in \mathbb{N}_0. \tag{28}$$

By using the change of variables

$$\begin{cases} x_n y_{n-1}z_{n-2}x_{n-3} = & v_n^{(1)}, \quad n \in \mathbb{N}_0, \\ y_n z_{n-1}x_{n-2}y_{n-3} = & \hat{v}_n^{(1)}, \quad n \in \mathbb{N}_0, \\ z_n x_{n-1}y_{n-2}z_{n-3} = & \tilde{v}_n^{(1)}, \quad n \geq -3, \end{cases} \tag{29}$$

system (28) becomes

$$v_n^{(1)} = \frac{1}{\beta}, \quad \hat{v}_n^{(1)} = \frac{1}{\theta}, \quad \tilde{v}_n^{(1)} = \frac{1}{\eta}\tilde{v}_{n-3}^{(1)}, \quad n \in \mathbb{N}_0. \tag{30}$$

From (2), the solution of the third equation in (30) is given

$$\tilde{v}_{3m+i}^{(1)} = \left(\frac{1}{\eta}\right)^{m+1} \tilde{v}_{i-3}^{(1)}, \tag{31}$$

for every $m \in \mathbb{N}_0$ and $i \in \{0, 1, 2\}$.
From (29) we gain

$$\begin{cases} x_n = & \frac{v_n^{(1)}\hat{v}_{n-4}^{(1)}\tilde{v}_{n-8}^{(1)}}{\hat{v}_{n-1}^{(1)}\tilde{v}_{n-5}^{(1)}v_{n-9}^{(1)}} x_{n-12}, \quad n \geq 9, \\ y_n = & \frac{\hat{v}_n^{(1)}\tilde{v}_{n-4}^{(1)}v_{n-8}^{(1)}}{\tilde{v}_{n-1}^{(1)}v_{n-5}^{(1)}\hat{v}_{n-9}^{(1)}} y_{n-12}, \quad n \geq 9, \\ z_n = & \frac{\tilde{v}_n^{(1)}v_{n-4}^{(1)}\hat{v}_{n-8}^{(1)}}{v_{n-1}^{(1)}\hat{v}_{n-5}^{(1)}\tilde{v}_{n-9}^{(1)}} z_{n-12}, \quad n \geq 6, \end{cases} \tag{32}$$

and eventually

$$
\begin{cases}
x_{12m+j_1} = & \dfrac{v^{(1)}_{12m+j_1}\hat{v}^{(1)}_{12m+j_1-4}\tilde{v}^{(1)}_{12m+j_1-8}}{\hat{v}^{(1)}_{12m+j_1-1}\tilde{v}^{(1)}_{12m+j_1-5}v^{(1)}_{12m+j_1-9}} x_{12(m-1)+j_1}, \\
y_{12m+j_1} = & \dfrac{\hat{v}^{(1)}_{12m+j_1}\tilde{v}^{(1)}_{12m+j_1-4}v^{(1)}_{12m+j_1-8}}{\tilde{v}^{(1)}_{12m+j_1-1}v^{(1)}_{12m+j_1-5}\hat{v}^{(1)}_{12m+j_1-9}} y_{12(m-1)+j_1}, \quad m \in \mathbb{N}_0, \\
z_{12m+j_2} = & \dfrac{\tilde{v}^{(1)}_{12m+j_2}v^{(1)}_{12m+j_2-4}\hat{v}^{(1)}_{12m+j_2-8}}{v^{(1)}_{12m+j_2-1}\hat{v}^{(1)}_{12m+j_2-5}\tilde{v}^{(1)}_{12m+j_2-9}} z_{12(m-1)+j_2},
\end{cases}
\tag{33}
$$

for $j_1 = \overline{9, 20}$ and $j_2 = \overline{6, 17}$.
From (33), we attain

$$
\begin{aligned}
x_{12m+3r_1+s} &= x_{3r_1+s-12} \prod_{p=0}^{m} \frac{v^{(1)}_{12p+3r_1+s}\hat{v}^{(1)}_{12p+3r_1+s-4}\tilde{v}^{(1)}_{12p+3r_1+s-8}}{\hat{v}^{(1)}_{12p+3r_1+s-1}\tilde{v}^{(1)}_{12p+3r_1+s-5}v^{(1)}_{12p+3r_1+s-9}}, \\
y_{12m+3r_1+s} &= y_{3r_1+s-12} \prod_{p=0}^{m} \frac{\hat{v}^{(1)}_{12p+3r_1+s}\tilde{v}^{(1)}_{12p+3r_1+s-4}v^{(1)}_{12p+3r_1+s-8}}{\tilde{v}^{(1)}_{12p+3r_1+s-1}v^{(1)}_{12p+3r_1+s-5}\hat{v}^{(1)}_{12p+3r_1+s-9}}, \\
z_{12m+3r_2+s} &= z_{3r_2+s-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(1)}_{12p+3r_2+s}v^{(1)}_{12p+3r_2+s-4}\hat{v}^{(1)}_{12p+3r_2+s-8}}{v^{(1)}_{12p+3r_2+s-1}\hat{v}^{(1)}_{12p+3r_2+s-5}\tilde{v}^{(1)}_{12p+3r_2+s-9}},
\end{aligned}
\tag{34}
$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3, 4, 5, 6\}$, $r_2 \in \{2, 3, 4, 5\}$ and $s \in \{0, 1, 2\}$.
From (31) and (34), we obtain

$$
x_{12m+3r_1+s} = \eta^{m+1} x_{3r_1+s-12}, \quad y_{12m+3r_1+s} = \eta^{m+1} y_{3r_1+s-12}, \quad z_{12m+3r_2+s} = \left(\frac{1}{\eta^3}\right)^{m+1} z_{3r_2+s-12}, \tag{35}
$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3, 4, 5, 6\}$, $r_2 \in \{2, 3, 4, 5\}$ and $s \in \{0, 1, 2\}$.

Case 4: Let $\beta = \gamma = \eta = 0$ and $\alpha\theta\zeta \neq 0$. In this case, we get the following system

$$
x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{\alpha y_{n-1}z_{n-2}}, \quad y_n = \frac{1}{\theta z_{n-1}x_{n-2}y_{n-3}}, \quad z_n = \frac{1}{\zeta x_{n-1}y_{n-2}z_{n-3}}, \quad n \in \mathbb{N}_0. \tag{36}
$$

By interchanging y_n, z_n, x_n, θ, ζ and α instead of x_n, y_n, z_n, β, θ and η in system (27), we obtain system (36). So, the solutions in (35) turn into the following formulas

$$
x_{12m+3r_2+s} = \left(\frac{1}{\alpha^3}\right)^{m+1} x_{3r_2+s-12}, \quad y_{12m+3r_1+s} = \alpha^{m+1} y_{3r_1+s-12}, \quad z_{12m+3r_1+s} = \alpha^{m+1} z_{3r_1+s-12},
$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3, 4, 5, 6\}$, $r_2 \in \{2, 3, 4, 5\}$ and $s \in \{0, 1, 2\}$.

Case 5: Let $\alpha = \theta = \eta = 0$ and $\beta\gamma\zeta \neq 0$. In this case, we get the following system

$$
x_n = \frac{1}{\beta y_{n-1}z_{n-2}x_{n-3}}, \quad y_n = \frac{z_{n-4}x_{n-5}y_{n-6}}{\gamma z_{n-1}x_{n-2}}, \quad z_n = \frac{1}{\zeta x_{n-1}y_{n-2}z_{n-3}}, \quad n \in \mathbb{N}_0. \tag{37}
$$

By interchanging z_n, x_n, y_n, ζ, β and γ instead of x_n, y_n, z_n, β, θ and η in system (27), we obtain system (37). So, the solutions in (35) turn into the following formulas

$$
x_{12m+3r_1+s} = \gamma^{m+1} x_{3r_1+s-12}, \quad y_{12m+3r_2+s} = \left(\frac{1}{\gamma^3}\right)^{m+1} y_{3r_2+s-12}, \quad z_{12m+3r_1+s} = \gamma^{m+1} z_{3r_1+s-12},
$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3, 4, 5, 6\}$, $r_2 \in \{2, 3, 4, 5\}$ and $s \in \{0, 1, 2\}$.

Case 6: Let $\alpha = \theta = \zeta = 0$ and $\beta\gamma\eta \neq 0$. In this case, system (12) transforms into the following system

$$x_n = \frac{1}{\beta y_{n-1} z_{n-2} x_{n-3}}, \quad y_n = \frac{z_{n-4} x_{n-5} y_{n-6}}{\gamma z_{n-1} x_{n-2}}, \quad z_n = \frac{x_{n-4} y_{n-5} z_{n-6}}{\eta x_{n-1} y_{n-2}}, \quad n \in \mathbb{N}_0. \tag{38}$$

Multiplying the first equation in system (38) by $y_{n-1}z_{n-2}x_{n-3}$ for all $n \in \mathbb{N}_0$, the second by $z_{n-1}x_{n-2}y_{n-3}$ for all $n \in \mathbb{N}_0$ and the third by $x_{n-1}y_{n-2}z_{n-3}$ for all $n \in \mathbb{N}_0$, it follows that

$$x_n y_{n-1} z_{n-2} x_{n-3} = \frac{1}{\beta}, \quad y_n z_{n-1} x_{n-2} y_{n-3} = \frac{y_{n-3} z_{n-4} x_{n-5} y_{n-6}}{\gamma}, \quad z_n x_{n-1} y_{n-2} z_{n-3} = \frac{z_{n-3} x_{n-4} y_{n-5} z_{n-6}}{\eta}, \tag{39}$$

where $n \in \mathbb{N}_0$.
By employing change of variables

$$\begin{cases} x_n y_{n-1} z_{n-2} x_{n-3} = & v_n^{(2)}, \quad n \in \mathbb{N}_0, \\ y_n z_{n-1} x_{n-2} y_{n-3} = & \hat{v}_n^{(2)}, \quad n \geq -3, \\ z_n x_{n-1} y_{n-2} z_{n-3} = & \tilde{v}_n^{(2)}, \quad n \geq -3, \end{cases} \tag{40}$$

system (39) becomes

$$v_n^{(2)} = \frac{1}{\beta}, \quad \hat{v}_n^{(2)} = \frac{1}{\gamma} \hat{v}_{n-3}^{(2)}, \quad \tilde{v}_n^{(2)} = \frac{1}{\eta} \tilde{v}_{n-3}^{(2)}, \quad n \in \mathbb{N}_0. \tag{41}$$

From (2), the solutions of the second and the third equations in (41) is given, respectively

$$\hat{v}_{3m+i}^{(2)} = \left(\frac{1}{\gamma}\right)^{m+1} \hat{v}_{i-3}^{(2)}, \quad \tilde{v}_{3m+i}^{(2)} = \left(\frac{1}{\eta}\right)^{m+1} \tilde{v}_{i-3}^{(2)}, \tag{42}$$

for every $m \in \mathbb{N}_0$ and $i \in \{0, 1, 2\}$.
From (40), we obtain

$$\begin{cases} x_n = & \dfrac{v_n^{(2)} \hat{v}_{n-4}^{(2)} \tilde{v}_{n-8}^{(2)}}{\hat{v}_{n-1}^{(2)} \tilde{v}_{n-5}^{(2)} v_{n-9}^{(2)}} x_{n-12}, \quad n \geq 9, \\ y_n = & \dfrac{\hat{v}_n^{(2)} \tilde{v}_{n-4}^{(2)} v_{n-8}^{(2)}}{\tilde{v}_{n-1}^{(2)} v_{n-5}^{(2)} \hat{v}_{n-9}^{(2)}} y_{n-12}, \quad n \geq 8, \\ z_n = & \dfrac{\tilde{v}_n^{(2)} v_{n-4}^{(2)} \hat{v}_{n-8}^{(2)}}{v_{n-1}^{(2)} \hat{v}_{n-5}^{(2)} \tilde{v}_{n-9}^{(2)}} z_{n-12}, \quad n \geq 6, \end{cases} \tag{43}$$

and eventually

$$\begin{cases} x_{12m+j_1} = & \dfrac{v_{12m+j_1}^{(2)} \hat{v}_{12m+j_1-4}^{(2)} \tilde{v}_{12m+j_1-8}^{(2)}}{\hat{v}_{12m+j_1-1}^{(2)} \tilde{v}_{12m+j_1-5}^{(2)} v_{12m+j_1-9}^{(2)}} x_{12(m-1)+j_1}, \\ y_{12m+j_2} = & \dfrac{\hat{v}_{12m+j_2}^{(2)} \tilde{v}_{12m+j_2-4}^{(2)} v_{12m+j_2-8}^{(2)}}{\tilde{v}_{12m+j_2-1}^{(2)} v_{12m+j_2-5}^{(2)} \hat{v}_{12m+j_2-9}^{(2)}} y_{12(m-1)+j_2}, \quad m \in \mathbb{N}_0, \\ z_{12m+j_3} = & \dfrac{\tilde{v}_{12m+j_3}^{(2)} v_{12m+j_3-4}^{(2)} \hat{v}_{12m+j_3-8}^{(2)}}{v_{12m+j_3-1}^{(2)} \hat{v}_{12m+j_3-5}^{(2)} \tilde{v}_{12m+j_3-9}^{(2)}} z_{12(m-1)+j_3}, \end{cases} \tag{44}$$

for $j_1 = \overline{9, 20}$, $j_2 = \overline{8, 19}$ and $j_3 = \overline{6, 17}$.
From (44), we attain

$$x_{12m+3r_1+s_1} = x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{v_{12p+3r_1+s_1}^{(2)} \hat{v}_{12p+3r_1+s_1-4}^{(2)} \tilde{v}_{12p+3r_1+s_1-8}^{(2)}}{\hat{v}_{12p+3r_1+s_1-1}^{(2)} \tilde{v}_{12p+3r_1+s_1-5}^{(2)} v_{12p+3r_1+s_1-9}^{(2)}},$$

$$y_{12m+3r_2+s_2} = y_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\hat{v}^{(2)}_{12p+3r_2+s_2}\tilde{v}^{(2)}_{12p+3r_2+s_2-4}v^{(2)}_{12p+3r_2+s_2-8}}{\tilde{v}^{(2)}_{12p+3r_2+s_2-1}v^{(2)}_{12p+3r_2+s_2-5}\hat{v}^{(2)}_{12p+3r_2+s_2-9}},$$
$$z_{12m+3r_2+s_1} = z_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(2)}_{12p+3r_2+s_1}v^{(2)}_{12p+3r_2+s_1-4}\hat{v}^{(2)}_{12p+3r_2+s_1-8}}{v^{(2)}_{12p+3r_2+s_1-1}\hat{v}^{(2)}_{12p+3r_2+s_1-5}\tilde{v}^{(2)}_{12p+3r_2+s_1-9}}, \tag{45}$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.
From (42) and (45), we obtain

$$x_{12m+3r_1+s_1} = (\gamma\eta)^{m+1}x_{3r_1+s_1-12},\quad y_{12m+3r_2+s_2} = \left(\frac{\eta}{\gamma^3}\right)^{m+1}y_{3r_2+s_2-12},\quad z_{12m+3r_2+s_1} = \left(\frac{\gamma}{\eta^3}\right)^{m+1}z_{3r_2+s_1-12}, \tag{46}$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case 7: Let $\beta = \gamma = \zeta = 0$ and $\alpha\theta\eta \neq 0$ In this case, we get the following system

$$x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{\alpha y_{n-1}z_{n-2}},\quad y_n = \frac{1}{\theta z_{n-1}x_{n-2}y_{n-3}},\quad z_n = \frac{x_{n-4}y_{n-5}z_{n-6}}{\eta x_{n-1}y_{n-2}},\quad n \in \mathbb{N}_0. \tag{47}$$

By interchanging y_n, z_n, x_n, θ, η and α instead of x_n, y_n, z_n, β, γ and η in system (38), we obtain system (47). So, the solutions in (46) turn into the following formulas

$$x_{12m+3r_2+s_1} = \left(\frac{\eta}{\alpha^3}\right)^{m+1}x_{3r_2+s_1-12},\quad y_{12m+3r_1+s_1} = (\alpha\eta)^{m+1}y_{3r_1+s_1-12},\quad z_{12m+3r_2+s_2} = \left(\frac{\alpha}{\eta^3}\right)^{m+1}z_{3r_2+s_2-12},$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case 8: Let $\beta = \theta = \eta = 0$ and $\alpha\gamma\zeta \neq 0$ In this case, we get the following system

$$x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{\alpha y_{n-1}z_{n-2}},\quad y_n = \frac{z_{n-4}x_{n-5}y_{n-6}}{\gamma z_{n-1}x_{n-2}},\quad z_n = \frac{1}{\zeta x_{n-1}y_{n-2}z_{n-3}},\quad n \in \mathbb{N}_0. \tag{48}$$

By interchanging z_n, x_n, y_n, ζ, α and γ instead of x_n, y_n, z_n, β, γ and η in system (38), we obtain system (48). So, the solutions in (46) turn into the formulas

$$x_{12m+3r_2+s_2} = \left(\frac{\gamma}{\alpha^3}\right)^{m+1}x_{3r_2+s_2-12},\quad y_{12m+3r_2+s_1} = \left(\frac{\alpha}{\gamma^3}\right)^{m+1}y_{3r_2+s_1-12},\quad z_{12m+3r_1+s_1} = (\alpha\gamma)^{m+1}z_{3r_1+s_1-12},$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case 9: Let $\alpha = \gamma = 0$ and $\beta\theta\eta\zeta \neq 0$. In this case, system (12) transforms into the following system

$$x_n = \frac{1}{\beta y_{n-1}z_{n-2}x_{n-3}},\quad y_n = \frac{1}{\theta z_{n-1}x_{n-2}y_{n-3}},\quad z_n = \frac{x_{n-4}y_{n-5}z_{n-6}}{x_{n-1}y_{n-2}\left(\eta + \zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}\right)},\quad n \in \mathbb{N}_0. \tag{49}$$

Multiplying the first equation in system (49) by $y_{n-1}z_{n-2}x_{n-3}$ for all $n \in \mathbb{N}_0$, the second by $z_{n-1}x_{n-2}y_{n-3}$ for all $n \in \mathbb{N}_0$ and the third by $x_{n-1}y_{n-2}z_{n-3}$ for all $n \in \mathbb{N}_0$, it follows that

$$x_ny_{n-1}z_{n-2}x_{n-3} = \frac{1}{\beta},\quad y_nz_{n-1}x_{n-2}y_{n-3} = \frac{1}{\theta},\quad z_nx_{n-1}y_{n-2}z_{n-3} = \frac{z_{n-3}x_{n-4}y_{n-5}z_{n-6}}{\eta + \zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}},\quad n \in \mathbb{N}_0. \tag{50}$$

By using the change of variables

$$\begin{cases} x_ny_{n-1}z_{n-2}x_{n-3} = v^{(3)}_n, & n \in \mathbb{N}_0, \\ y_nz_{n-1}x_{n-2}y_{n-3} = \hat{v}^{(3)}_n, & n \in \mathbb{N}_0, \\ z_nx_{n-1}y_{n-2}z_{n-3} = \frac{1}{\tilde{v}^{(3)}_n}, & n \geq -3, \end{cases} \tag{51}$$

system (50) becomes

$$v_n^{(3)} = \frac{1}{\beta}, \quad \hat{v}_n^{(3)} = \frac{1}{\theta}, \quad \tilde{v}_n^{(3)} = \eta \tilde{v}_{n-3}^{(3)} + \zeta, \quad n \in \mathbb{N}_0. \tag{52}$$

From (4), the solution of the third equation in (52) is given

$$\tilde{v}_{3m+i}^{(3)} = \begin{cases} \frac{\zeta+\eta^{m+1}\left((1-\eta)\tilde{v}_{i-3}^{(3)}-\zeta\right)}{1-\eta}, & \eta \neq 1, \\ \tilde{v}_{i-3}^{(3)} + \zeta\,(m+1)\,, & \eta = 1, \end{cases} \tag{53}$$

where $m \in \mathbb{N}_0$ and $i \in \{0,1,2\}$.
From (51), we gain

$$\begin{cases} x_n = \frac{v_n^{(3)}\hat{v}_{n-4}^{(3)}\tilde{v}_{n-5}^{(3)}}{\hat{v}_{n-1}^{(3)}\tilde{v}_{n-8}^{(3)}v_{n-9}^{(3)}} x_{n-12}, & n \geq 9, \\ y_n = \frac{\hat{v}_n^{(3)}\tilde{v}_{n-1}^{(3)}v_{n-8}^{(3)}}{\tilde{v}_{n-4}^{(3)}v_{n-5}^{(3)}\hat{v}_{n-9}^{(3)}} y_{n-12}, & n \geq 9, \\ z_n = \frac{v_{n-4}^{(3)}\hat{v}_{n-8}^{(3)}\tilde{v}_{n-9}^{(3)}}{\tilde{v}_n^{(3)}v_{n-1}^{(3)}\hat{v}_{n-5}^{(3)}} z_{n-12}, & n \geq 8, \end{cases} \tag{54}$$

and eventually

$$\begin{cases} x_{12m+j_1} = \frac{v_{12m+j_1}^{(3)}\hat{v}_{12m+j_1-4}^{(3)}\tilde{v}_{12m+j_1-5}^{(3)}}{\hat{v}_{12m+j_1-1}^{(3)}\tilde{v}_{12m+j_1-8}^{(3)}v_{12m+j_1-9}^{(3)}} x_{12(m-1)+j_1}, \\ y_{12m+j_1} = \frac{\hat{v}_{12m+j_1}^{(3)}\tilde{v}_{12m+j_1-1}^{(3)}v_{12m+j_1-8}^{(3)}}{\tilde{v}_{12m+j_1-4}^{(3)}v_{12m+j_1-5}^{(3)}\hat{v}_{12m+j_1-9}^{(3)}} y_{12(m-1)+j_1}, \\ z_{12m+j_2} = \frac{v_{12m+j_2-4}^{(3)}\hat{v}_{12m+j_2-8}^{(3)}\tilde{v}_{12m+j_2-9}^{(3)}}{\tilde{v}_{12m+j_2}^{(3)}v_{12m+j_2-1}^{(3)}\hat{v}_{12m+j_2-5}^{(3)}} z_{12(m-1)+j_2}, \end{cases} \tag{55}$$

for $m \in \mathbb{N}_0$, $j_1 = \overline{9,20}$ and $j_2 = \overline{8,19}$.
From (55), we attain

$$\begin{aligned} x_{12m+3r_1+s_1} &= x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{v_{12p+3r_1+s_1}^{(3)}\hat{v}_{12p+3r_1+s_1-4}^{(3)}\tilde{v}_{12p+3r_1+s_1-5}^{(3)}}{\hat{v}_{12p+3r_1+s_1-1}^{(3)}\tilde{v}_{12p+3r_1+s_1-8}^{(3)}v_{12p+3r_1+s_1-9}^{(3)}}, \\ y_{12m+3r_1+s_1} &= y_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\hat{v}_{12p+3r_1+s_1}^{(3)}\tilde{v}_{12p+3r_1+s_1-1}^{(3)}v_{12p+3r_1+s_1-8}^{(3)}}{\tilde{v}_{12p+3r_1+s_1-4}^{(3)}v_{12p+3r_1+s_1-5}^{(3)}\hat{v}_{12p+3r_1+s_1-9}^{(3)}}, \\ z_{12m+3r_2+s_2} &= z_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{v_{12p+3r_2+s_2-4}^{(3)}\hat{v}_{12p+3r_2+s_2-8}^{(3)}\tilde{v}_{12p+3r_2+s_2-9}^{(3)}}{\tilde{v}_{12p+3r_2+s_2}^{(3)}v_{12p+3r_2+s_2-1}^{(3)}\hat{v}_{12p+3r_2+s_2-5}^{(3)}}, \end{aligned} \tag{56}$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.
From (53) and (56), we obtain

$$\begin{aligned} x_{12m+3r_1+s_1} &= x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r_1+\lfloor \frac{s_1-2}{3} \rfloor}\left((1-\eta)\tilde{v}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor}^{(3)} - \zeta\right)}{\zeta + \eta^{4p+r_1-1+\lfloor \frac{s_1-2}{3} \rfloor}\left((1-\eta)\tilde{v}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor}^{(3)} - \zeta\right)}, \\ y_{12m+3r_1+s_1} &= y_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r_1+1+\lfloor \frac{s_1-1}{3} \rfloor}\left((1-\eta)\tilde{v}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor}^{(3)} - \zeta\right)}{\zeta + \eta^{4p+r_1+\lfloor \frac{s_1-1}{3} \rfloor}\left((1-\eta)\tilde{v}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor}^{(3)} - \zeta\right)}, \end{aligned} \tag{57}$$

$$z_{12m+3r_2+s_2} = z_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\eta)\tilde{v}^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} - \zeta\right)}{\zeta + \eta^{4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\eta)\tilde{v}^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} - \zeta\right)},$$

if $\eta \neq 1$, and

$$\begin{aligned} x_{12m+3r_1+s_1} &= x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(3)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} + \zeta\left(4p + r_1 + \lfloor\frac{s_1-2}{3}\rfloor\right)}{\tilde{v}^{(3)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} + \zeta\left(4p + r_1 - 1 + \lfloor\frac{s_1-2}{3}\rfloor\right)}, \\ y_{12m+3r_1+s_1} &= y_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(3)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} + \zeta\left(4p + r_1 + 1 + \lfloor\frac{s_1-1}{3}\rfloor\right)}{\tilde{v}^{(3)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} + \zeta\left(4p + r_1 + \lfloor\frac{s_1-1}{3}\rfloor\right)}, \\ z_{12m+3r_2+s_2} &= z_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} + \zeta\left(4p + r_2 - 1 + \lfloor\frac{s_2-3}{3}\rfloor\right)}{\tilde{v}^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} + \zeta\left(4p + r_2 + 2 + \lfloor\frac{s_2-3}{3}\rfloor\right)}, \end{aligned} \tag{58}$$

if $\eta = 1$, where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case10: Let $\gamma = \eta = 0$ and $\alpha\beta\theta\zeta \neq 0$. In this case, we get the following system

$$x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{y_{n-1}z_{n-2}\left(\alpha + \beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}\right)}, \quad y_n = \frac{1}{\theta z_{n-1}x_{n-2}y_{n-3}}, \quad z_n = \frac{1}{\zeta x_{n-1}y_{n-2}z_{n-3}}, \quad n \in \mathbb{N}_0. \tag{59}$$

By interchanging y_n, z_n, x_n, θ, ζ, β and α instead of x_n, y_n, z_n, β, θ, ζ and η in system (49), we obtain system (59). So, the solutions in (57) and (58) turn into the following formulas

$$\begin{aligned} x_{12m+3r_2+s_2} &= x_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\alpha)v^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} - \beta\right)}{\beta + \alpha^{4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\alpha)v^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} - \beta\right)}, \\ y_{12m+3r_1+s_1} &= y_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\alpha)v^{(3)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \beta\right)}{\beta + \alpha^{4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\alpha)v^{(3)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \beta\right)}, \\ z_{12m+3r_1+s_1} &= z_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\alpha)v^{(3)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} - \beta\right)}{\beta + \alpha^{4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\alpha)v^{(3)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} - \beta\right)}, \end{aligned}$$

if $\alpha \neq 1$, and

$$\begin{aligned} x_{12m+3r_2+s_2} &= x_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{v^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} + \beta\left(4p + r_2 - 1 + \lfloor\frac{s_2-3}{3}\rfloor\right)}{v^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} + \beta\left(4p + r_2 + 2 + \lfloor\frac{s_2-3}{3}\rfloor\right)}, \\ y_{12m+3r_1+s_1} &= y_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{v^{(3)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} + \beta\left(4p + r_1 + \lfloor\frac{s_1-2}{3}\rfloor\right)}{v^{(3)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} + \beta\left(4p + r_1 - 1 + \lfloor\frac{s_1-2}{3}\rfloor\right)}, \end{aligned}$$

$$z_{12m+3r_1+s_1} = z_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{v^{(3)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} + \beta\left(4p + r_1 + 1 + \lfloor\frac{s_1-1}{3}\rfloor\right)}{v^{(3)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} + \beta\left(4p + r_1 + \lfloor\frac{s_1-1}{3}\rfloor\right)},$$

if $\alpha = 1$, where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case11: Let $\alpha = \eta = 0$ and $\beta\gamma\theta\zeta \neq 0$. In this case, we get the following system

$$x_n = \frac{1}{\beta y_{n-1}z_{n-2}x_{n-3}}, \quad y_n = \frac{z_{n-4}x_{n-5}y_{n-6}}{z_{n-1}x_{n-2}\left(\gamma + \theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}\right)}, \quad z_n = \frac{1}{\zeta x_{n-1}y_{n-2}z_{n-3}}, \quad n \in \mathbb{N}_0. \tag{60}$$

By interchanging z_n, x_n, y_n, ζ, β, θ and γ instead of x_n, y_n, z_n, β, θ, ζ and η in system (49), we obtain system (60). So, the solutions in (57) and (58) turn into the following formulas

$$x_{12m+3r_1+s_1} = x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(3)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} - \theta\right)}{\theta + \gamma^{4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(3)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} - \theta\right)},$$

$$y_{12m+3r_2+s_2} = y_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\gamma)\hat{v}^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} - \theta\right)}{\theta + \gamma^{4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\gamma)\hat{v}^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} - \theta\right)},$$

$$z_{12m+3r_1+s_1} = z_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(3)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \theta\right)}{\theta + \gamma^{4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(3)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \theta\right)}$$

if $\gamma \neq 1$, and

$$x_{12m+3r_1+s_1} = x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\hat{v}^{(3)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} + \theta\left(4p + r_1 + 1 + \lfloor\frac{s_1-1}{3}\rfloor\right)}{\hat{v}^{(3)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} + \theta\left(4p + r_1 + \lfloor\frac{s_1-1}{3}\rfloor\right)},$$

$$y_{12m+3r_2+s_2} = y_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\hat{v}^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} + \theta\left(4p + r_2 - 1 + \lfloor\frac{s_2-3}{3}\rfloor\right)}{\hat{v}^{(3)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} + \theta\left(4p + r_2 + 2 + \lfloor\frac{s_2-3}{3}\rfloor\right)},$$

$$z_{12m+3r_1+s_1} = z_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\hat{v}^{(3)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} + \theta\left(4p + r_1 + \lfloor\frac{s_1-2}{3}\rfloor\right)}{\hat{v}^{(3)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} + \theta\left(4p + r_1 - 1 + \lfloor\frac{s_1-2}{3}\rfloor\right)},$$

if $\gamma = 1$, where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case:12 Let $\beta = \theta = 0$ and $\alpha\gamma\eta\zeta \neq 0$. In this case, system (12) turns into the following system

$$x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{\alpha y_{n-1}z_{n-2}}, \quad y_n = \frac{z_{n-4}x_{n-5}y_{n-6}}{\gamma z_{n-1}x_{n-2}}, \quad z_n = \frac{x_{n-4}y_{n-5}z_{n-6}}{x_{n-1}y_{n-2}\left(\eta + \zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}\right)}, \quad n \in \mathbb{N}_0. \tag{61}$$

Multiplying the first equation in system (61) by $y_{n-1}z_{n-2}x_{n-3}$ for all $n \in \mathbb{N}_0$, the second by $z_{n-1}x_{n-2}y_{n-3}$ for all $n \in \mathbb{N}_0$ and the third by $x_{n-1}y_{n-2}z_{n-3}$ for all $n \in \mathbb{N}_0$, it follows that

$$x_n y_{n-1}z_{n-2}x_{n-3} = \frac{x_{n-3}y_{n-4}z_{n-5}x_{n-6}}{\alpha}, \quad y_n z_{n-1}x_{n-2}y_{n-3} = \frac{y_{n-3}z_{n-4}x_{n-5}y_{n-6}}{\gamma}, \quad z_n x_{n-1}y_{n-2}z_{n-3} = \frac{z_{n-3}x_{n-4}y_{n-5}z_{n-6}}{\eta + \zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}},$$

(62)

for $n \in \mathbb{N}_0$.
By using the change of variables

$$x_n y_{n-1} z_{n-2} x_{n-3} = v_n^{(4)}, \quad y_n z_{n-1} x_{n-2} y_{n-3} = \hat{v}_n^{(4)}, \quad z_n x_{n-1} y_{n-2} z_{n-3} = \frac{1}{\tilde{v}_n^{(4)}}, \quad n \geq -3, \tag{63}$$

system (62) becomes

$$v_n^{(4)} = \frac{1}{\alpha} v_{n-3}^{(4)}, \quad \hat{v}_n^{(4)} = \frac{1}{\gamma} \hat{v}_{n-3}^{(4)}, \quad \tilde{v}_n^{(4)} = \eta \tilde{v}_{n-3}^{(4)} + \zeta, \quad n \in \mathbb{N}_0. \tag{64}$$

From (2), the solutions of the first and the second equations in (64) and from (4), the solution of the third equation in (64) is given respectively

$$\begin{cases} v_{3m+i}^{(4)} = & \left(\frac{1}{\alpha}\right)^{m+1} v_{i-3}^{(4)}, \\ \hat{v}_{3m+i}^{(4)} = & \left(\frac{1}{\gamma}\right)^{m+1} \hat{v}_{i-3}^{(4)}, \\ \tilde{v}_{3m+i}^{(4)} = & \begin{cases} \frac{\zeta + \eta^{m+1}\left((1-\eta)\tilde{v}_{i-3}^{(4)} - \zeta\right)}{1-\eta}, & \eta \neq 1, \\ \tilde{v}_m^{(4)} + \zeta\,(m+1), & \eta = 1, \end{cases} \end{cases} \tag{65}$$

where $m \in \mathbb{N}_0$, and $i \in \{0, 1, 2\}$.
From (63), we obtain

$$x_n = \frac{v_n^{(4)} \hat{v}_{n-4}^{(4)} \tilde{v}_{n-5}^{(4)}}{\hat{v}_{n-1}^{(4)} \tilde{v}_{n-8}^{(4)} v_{n-9}^{(4)}} x_{n-12}, \quad y_n = \frac{\hat{v}_n^{(4)} \tilde{v}_{n-1}^{(4)} v_{n-8}^{(4)}}{\tilde{v}_{n-4}^{(4)} v_{n-5}^{(4)} \hat{v}_{n-9}^{(4)}} y_{n-12}, \quad z_n = \frac{v_{n-4}^{(4)} \hat{v}_{n-8}^{(4)} \tilde{v}_{n-9}^{(4)}}{\tilde{v}_n^{(4)} v_{n-1}^{(4)} \hat{v}_{n-5}^{(4)}} z_{n-12}, \quad n \geq 6, \tag{66}$$

and consequently

$$\begin{cases} x_{12m+j} = & \frac{v_{12m+j}^{(4)} \hat{v}_{12m+j-4}^{(4)} \tilde{v}_{12m+j-5}^{(4)}}{\hat{v}_{12m+j-1}^{(4)} \tilde{v}_{12m+j-8}^{(4)} v_{12m+j-9}^{(4)}} x_{12(m-1)+j}, \\ y_{12m+j} = & \frac{\hat{v}_{12m+j}^{(4)} \tilde{v}_{12m+j-1}^{(4)} v_{12m+j-8}^{(4)}}{\tilde{v}_{12m+j-4}^{(4)} v_{12m+j-5}^{(4)} \hat{v}_{12m+j-9}^{(4)}} y_{12(m-1)+j}, \\ z_{12m+j} = & \frac{v_{12m+j-4}^{(4)} \hat{v}_{12m+j-8}^{(4)} \tilde{v}_{12m+j-9}^{(4)}}{\tilde{v}_{12m+j}^{(4)} v_{12m+j-1}^{(4)} \hat{v}_{12m+j-5}^{(4)}} z_{12(m-1)+j}, \end{cases} \tag{67}$$

where $m \in \mathbb{N}_0$ and $j = \overline{6, 17}$.
From (67), we attain

$$\begin{aligned} x_{12m+3r+s} =& x_{3r+s-12} \prod_{p=0}^{m} \frac{v_{12p+3r+s}^{(4)} \hat{v}_{12p+3r+s-4}^{(4)} \tilde{v}_{12p+3r+s-5}^{(4)}}{\hat{v}_{12p+3r+s-1}^{(4)} \tilde{v}_{12p+3r+s-8}^{(4)} v_{12p+3r+s-9}^{(4)}}, \\ y_{12m+3r+s} =& y_{3r+s-12} \prod_{p=0}^{m} \frac{\hat{v}_{12p+3r+s}^{(4)} \tilde{v}_{12p+3r+s-1}^{(4)} v_{12p+3r+s-8}^{(4)}}{\tilde{v}_{12p+3r+s-4}^{(4)} v_{12p+3r+s-5}^{(4)} \hat{v}_{12p+3r+s-9}^{(4)}}, \\ z_{12m+3r+s} =& z_{3r+s-12} \prod_{p=0}^{m} \frac{v_{12p+3r+s-4}^{(4)} \hat{v}_{12p+3r+s-8}^{(4)} \tilde{v}_{12p+3r+s-9}^{(4)}}{\tilde{v}_{12p+3r+s}^{(4)} v_{12p+3r+s-1}^{(4)} \hat{v}_{12p+3r+s-5}^{(4)}}, \end{aligned} \tag{68}$$

where $m \in \mathbb{N}_0$, $r \in \{2, 3, 4, 5\}$ and $s \in \{0, 1, 2\}$.
From (65) and (68), we gain

$$x_{12m+3r+s} = \left(\frac{\gamma}{\alpha^3}\right)^{m+1} x_{3r+s-12} \prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r+\lfloor \frac{s-2}{3} \rfloor} \left((1-\eta)\tilde{v}_{s-5-3\lfloor \frac{s-2}{3} \rfloor}^{(4)} - \zeta\right)}{\zeta + \eta^{4p+r-1+\lfloor \frac{s-2}{3} \rfloor} \left((1-\eta)\tilde{v}_{s-5-3\lfloor \frac{s-2}{3} \rfloor}^{(4)} - \zeta\right)},$$

$$y_{12m+3r+s}=\left(\frac{\alpha}{\gamma^3}\right)^{m+1}y_{3r+s-12}\prod_{p=0}^{m}\frac{\zeta+\eta^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(4)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\zeta\right)}{\zeta+\eta^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(4)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\zeta\right)},$$
$$z_{12m+3r+s}=(\alpha\gamma)^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{\zeta+\eta^{4p+r-2}\left((1-\eta)\tilde{v}^{(4)}_{s-3}-\zeta\right)}{\zeta+\eta^{4p+r+1}\left((1-\eta)\tilde{v}^{(4)}_{s-3}-\zeta\right)}, \tag{69}$$

if $\eta\neq 1$, and

$$x_{12m+3r+s}=\left(\frac{\gamma}{\alpha^3}\right)^{m+1}x_{3r+s-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(4)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\tilde{v}^{(4)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$
$$y_{12m+3r+s}=\left(\frac{\alpha}{\gamma^3}\right)^{m+1}y_{3r+s-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(4)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\tilde{v}^{(4)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}, \tag{70}$$
$$z_{12m+3r+s}=(\alpha\gamma)^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(4)}_{s-3}+\zeta(4p+r-2)}{\tilde{v}^{(4)}_{s-3}+\zeta(4p+r+1)},$$

if $\eta=1$, where $m\in\mathbb{N}_0$, $r\in\{2,3,4,5\}$ and $s\in\{0,1,2\}$.

Case 13: Let $\theta=\zeta=0$ and $\alpha\beta\gamma\eta\neq 0$. In this case, we get the following system

$$x_n=\frac{y_{n-4}z_{n-5}x_{n-6}}{y_{n-1}z_{n-2}\left(\alpha+\beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}\right)},\quad y_n=\frac{z_{n-4}x_{n-5}y_{n-6}}{\gamma z_{n-1}x_{n-2}},\quad z_n=\frac{x_{n-4}y_{n-5}z_{n-6}}{\eta x_{n-1}y_{n-2}},\quad n\in\mathbb{N}_0. \tag{71}$$

By interchanging y_n, z_n, x_n, γ, η, β and α instead of x_n, y_n, z_n, α, γ, ζ and η in system (61), we obtain system (71). So, the solutions in (69) and (70) turn into the following formulas

$$x_{12m+3r+s}=(\gamma\eta)^{m+1}x_{3r+s-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r-2}\left((1-\alpha)v^{(4)}_{s-3}-\beta\right)}{\beta+\alpha^{4p+r+1}\left((1-\alpha)v^{(4)}_{s-3}-\beta\right)},$$
$$y_{12m+3r+s}=\left(\frac{\eta}{\gamma^3}\right)^{m+1}y_{3r+s-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(4)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\beta\right)}{\beta+\alpha^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(4)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\beta\right)},$$
$$z_{12m+3r+s}=\left(\frac{\gamma}{\eta^3}\right)^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(4)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\beta\right)}{\beta+\alpha^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(4)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\beta\right)},$$

if $\alpha\neq 1$, and

$$x_{12m+3r+s}=(\gamma\eta)^{m+1}x_{3r+s-12}\prod_{p=0}^{m}\frac{v^{(4)}_{s-3}+\beta(4p+r-2)}{v^{(4)}_{s-3}+\beta(4p+r+1)},$$
$$y_{12m+3r+s}=\left(\frac{\eta}{\gamma^3}\right)^{m+1}y_{3r+s-12}\prod_{p=0}^{m}\frac{v^{(4)}_{s-5-3\rfloor\frac{s-2}{3}\rfloor}+\beta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{v^{(4)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\beta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$
$$z_{12m+3r+s}=\left(\frac{\gamma}{\eta^3}\right)^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{v^{(4)}_{s-4-3\rfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{v^{(4)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)},$$

if $\alpha = 1$, where $m \in \mathbb{N}_0$, $r \in \{2, 3, 4, 5\}$ and $s \in \{0, 1, 2\}$.

Case 14: Let $\beta = \zeta = 0$ and $\alpha\gamma\theta\eta \neq 0$. In this case, we get the following system

$$x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{\alpha y_{n-1}z_{n-2}}, \quad y_n = \frac{z_{n-4}x_{n-5}y_{n-6}}{z_{n-1}x_{n-2}\left(\gamma + \theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}\right)}, \quad z_n = \frac{x_{n-4}y_{n-5}z_{n-6}}{\eta x_{n-1}y_{n-2}}, \quad n \in \mathbb{N}_0. \tag{72}$$

By interchanging z_n, x_n, y_n, η, α, θ and γ instead of x_n, y_n, z_n, α, γ, ζ and η in system (61), we obtain system (72). So, the solutions in (69) and (70) turn into the following formulas

$$x_{12m+3r+s} = \left(\frac{\eta}{\alpha^3}\right)^{m+1} x_{3r+s-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{\vartheta}^{(4)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \theta\right)}{\theta + \gamma^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{\vartheta}^{(4)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \theta\right)},$$

$$y_{12m+3r+s} = (\alpha\eta)^{m+1} y_{3r+s-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r-2}\left((1-\gamma)\hat{\vartheta}^{(4)}_{s-3} - \theta\right)}{\theta + \gamma^{4p+r+1}\left((1-\gamma)\hat{\vartheta}^{(4)}_{s-3} - \theta\right)},$$

$$z_{12m+3r+s} = \left(\frac{\alpha}{\eta^3}\right)^{m+1} z_{3r+s-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{\vartheta}^{(4)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)}{\theta + \gamma^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{\vartheta}^{(4)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)},$$

if $\gamma \neq 1$, and

$$x_{12m+3r+s} = \left(\frac{\eta}{\alpha^3}\right)^{m+1} x_{3r+s-12} \prod_{p=0}^{m} \frac{\hat{\vartheta}^{(4)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \theta\left(4p + r + 1 + \lfloor\frac{s-1}{3}\rfloor\right)}{\hat{\vartheta}^{(4)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \theta\left(4p + r + \lfloor\frac{s-1}{3}\rfloor\right)},$$

$$y_{12m+3r+s} = (\alpha\eta)^{m+1} y_{3r+s-12} \prod_{p=0}^{m} \frac{\hat{\vartheta}^{(4)}_{s-3} + \theta\left(4p + r - 2\right)}{\hat{\vartheta}^{(4)}_{s-3} + \theta\left(4p + r + 1\right)},$$

$$z_{12m+3r+s} = \left(\frac{\alpha}{\eta^3}\right)^{m+1} z_{3r+s-12} \prod_{p=0}^{m} \frac{\hat{\vartheta}^{(4)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \theta\left(4p + r + \lfloor\frac{s-2}{3}\rfloor\right)}{\hat{\vartheta}^{(4)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \theta\left(4p + r - 1 + \lfloor\frac{s-2}{3}\rfloor\right)},$$

if $\gamma = 1$, where $m \in \mathbb{N}_0$, $r \in \{2, 3, 4, 5\}$ and $s \in \{0, 1, 2\}$.

Case 15: Let $\alpha = \theta = 0$ and $\beta\gamma\eta\theta \neq 0$. In this case, system (12) transforms into the following system

$$x_n = \frac{1}{\beta y_{n-1}z_{n-2}x_{n-3}}, \quad y_n = \frac{z_{n-4}x_{n-5}y_{n-6}}{\gamma z_{n-1}x_{n-2}}, \quad z_n = \frac{x_{n-4}y_{n-5}z_{n-6}}{x_{n-1}y_{n-2}\left(\eta + \zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}\right)}, \quad n \in \mathbb{N}_0. \tag{73}$$

Multiplying the first equation system (73) by $y_{n-1}z_{n-2}x_{n-3}$ for all $n \in \mathbb{N}_0$, the second by $z_{n-1}x_{n-2}y_{n-3}$ for all $n \in \mathbb{N}_0$ and the third by $x_{n-1}y_{n-2}z_{n-3}$ for all $n \in \mathbb{N}_0$, it follows that

$$x_n y_{n-1}z_{n-2}x_{n-3} = \frac{1}{\beta}, \quad y_n z_{n-1}x_{n-2}y_{n-3} = \frac{y_{n-3}z_{n-4}x_{n-5}y_{n-6}}{\gamma}, \quad z_n x_{n-1}y_{n-2}z_{n-3} = \frac{z_{n-3}x_{n-4}y_{n-5}z_{n-6}}{\eta + \zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}}, \quad n \in \mathbb{N}_0. \tag{74}$$

By employing the change of variables

$$\begin{cases} x_n y_{n-1}z_{n-2}x_{n-3} = & v^{(5)}_n, \quad n \in \mathbb{N}_0, \\ y_n z_{n-1}x_{n-2}y_{n-3} = & \hat{\vartheta}^{(5)}_n, \quad n \geq -3, \\ z_n x_{n-1}y_{n-2}z_{n-3} = & \frac{1}{\hat{\vartheta}^{(5)}_n}, \quad n \geq -3, \end{cases} \tag{75}$$

system (74) becomes

$$v_n^{(5)} = \frac{1}{\beta}, \quad \hat{v}_n^{(5)} = \frac{1}{\gamma}\hat{v}_{n-3}^{(5)}, \quad \tilde{v}_n^{(5)} = \eta\tilde{v}_{n-3}^{(5)} + \zeta, \quad n \in \mathbb{N}_0. \tag{76}$$

From (2), the solution of the second equation in (76) and from (4), the solution of the third equation in (76) is given respectively

$$\begin{cases} \hat{v}_{3m+i}^{(5)} = & \left(\frac{1}{\gamma}\right)^{m+1} \hat{v}_{i-3}^{(5)}, \\ \tilde{v}_{3m+i}^{(5)} = & \begin{cases} \frac{\zeta+\eta^{m+1}\left((1-\eta)\tilde{v}_{i-3}^{(5)}-\zeta\right)}{1-\eta}, & \eta \neq 1, \\ \tilde{v}_{i-3}^{(5)} + \zeta\,(m+1)\,, & \eta = 1, \end{cases} \end{cases} \tag{77}$$

where $m \in \mathbb{N}_0$ and $i \in \{0, 1, 2\}$.
From (75), we gain

$$\begin{cases} x_n = & \frac{v_n^{(5)}\hat{v}_{n-4}^{(5)}\tilde{v}_{n-5}^{(5)}}{\hat{v}_{n-1}^{(5)}\tilde{v}_{n-8}^{(5)}v_{n-9}^{(5)}} x_{n-12}, \quad n \geq 9, \\ y_n = & \frac{\hat{v}_n^{(5)}\tilde{v}_{n-1}^{(5)}v_{n-8}^{(5)}}{\tilde{v}_{n-4}^{(5)}v_{n-5}^{(5)}\hat{v}_{n-9}^{(5)}} y_{n-12}, \quad n \geq 8, \\ z_n = & \frac{v_{n-4}^{(5)}\hat{v}_{n-8}^{(5)}\tilde{v}_{n-9}^{(5)}}{\tilde{v}_n^{(5)}v_{n-1}^{(5)}\hat{v}_{n-5}^{(5)}} z_{n-12}, \quad n \geq 6, \end{cases} \tag{78}$$

and consequently

$$\begin{cases} x_{12m+j_1} = & \frac{v_{12m+j_1}^{(5)}\hat{v}_{12m+j_1-4}^{(5)}\tilde{v}_{12m+j_1-5}^{(5)}}{\hat{v}_{12m+j_1-1}^{(5)}\tilde{v}_{12m+j_1-8}^{(5)}v_{12m+j_1-9}^{(5)}} x_{12(m-1)+j_1}, \\ y_{12m+j_2} = & \frac{\hat{v}_{12m+j_2}^{(5)}\tilde{v}_{12m+j_2-1}^{(5)}v_{12m+j_2-8}^{(5)}}{\tilde{v}_{12m+j_2-4}^{(5)}v_{12m+j_2-5}^{(5)}\hat{v}_{12m+j_2-9}^{(5)}} y_{12(m-1)+j_2}, \\ z_{12m+j_3} = & \frac{v_{12m+j_3-4}^{(5)}\hat{v}_{12m+j_3-8}^{(5)}\tilde{v}_{12m+j_3-9}^{(5)}}{\tilde{v}_{12m+j_3}^{(5)}v_{12m+j_3-1}^{(5)}\hat{v}_{12m+j_3-5}^{(5)}} z_{12(m-1)+j_3}, \end{cases} \tag{79}$$

where $m \in \mathbb{N}_0$, $j_1 = \overline{9, 20}$, $j_2 = \overline{8, 19}$ and $j_3 = \overline{6, 17}$.
From (79), we attain

$$\begin{aligned} x_{12m+3r_1+s_1} =& x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{v_{12p+3r_1+s_1}^{(5)}\hat{v}_{12p+3r_1+s_1-4}^{(5)}\tilde{v}_{12p+3r_1+s_1-5}^{(5)}}{\hat{v}_{12p+3r_1+s_1-1}^{(5)}\tilde{v}_{12p+3r_1+s_1-8}^{(5)}v_{12p+3r_1+s_1-9}^{(5)}}, \\ y_{12m+3r_2+s_2} =& y_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\hat{v}_{12p+3r_2+s_2}^{(5)}\tilde{v}_{12p+3r_2+s_2-1}^{(5)}v_{12p+3r_2+s_2-8}^{(5)}}{\tilde{v}_{12p+3r_2+s_2-4}^{(5)}v_{12p+3r_2+s_2-5}^{(5)}\hat{v}_{12p+3r_2+s_2-9}^{(5)}}, \\ z_{12m+3r_2+s_1} =& z_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{v_{12p+3r_2+s_1-4}^{(5)}\hat{v}_{12p+3r_2+s_1-8}^{(5)}\tilde{v}_{12p+3r_2+s_1-9}^{(5)}}{\tilde{v}_{12p+3r_2+s_1}^{(5)}v_{12p+3r_2+s_1-1}^{(5)}\hat{v}_{12p+3r_2+s_1-5}^{(5)}}, \end{aligned} \tag{80}$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3, 4, 5, 6\}$, $r_2 \in \{2, 3, 4, 5\}$, $s_1 \in \{0, 1, 2\}$ and $s_2 \in \{2, 3, 4\}$.
From (77) and (80), we obtain

$$x_{12m+3r_1+s_1} = \gamma^{m+1} x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r_1+\lfloor \frac{s_1-2}{3} \rfloor}\left((1-\eta)\tilde{v}^{(5)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} - \zeta\right)}{\zeta + \eta^{4p+r_1-1+\lfloor \frac{s_1-2}{3} \rfloor}\left((1-\eta)\tilde{v}^{(5)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} - \zeta\right)},$$

$$y_{12m+3r_2+s_2} = \left(\frac{1}{\gamma^3}\right)^{m+1} y_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r_2+1+\lfloor \frac{s_2-1}{3} \rfloor}\left((1-\eta)\tilde{v}^{(5)}_{s_2-4-3\lfloor \frac{s_2-1}{3} \rfloor} - \zeta\right)}{\zeta + \eta^{4p+r_2+\lfloor \frac{s_2-1}{3} \rfloor}\left((1-\eta)\tilde{v}^{(5)}_{s_2-4-3\lfloor \frac{s_2-1}{3} \rfloor} - \zeta\right)}, \tag{81}$$

$$z_{12m+3r_2+s_1} = \gamma^{m+1} z_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r_2-2}\left((1-\eta)\tilde{v}^{(5)}_{s_1-3} - \zeta\right)}{\zeta + \eta^{4p+r_2+1}\left((1-\eta)\tilde{v}^{(5)}_{s_1-3} - \zeta\right)},$$

if $\eta \neq 1$, and

$$x_{12m+3r_1+s_1} = \gamma^{m+1} x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(5)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} + \zeta\left(4p + r_1 + \lfloor \frac{s_1-2}{3} \rfloor\right)}{\tilde{v}^{(5)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} + \zeta\left(4p + r_1 - 1 + \lfloor \frac{s_1-2}{3} \rfloor\right)},$$

$$y_{12m+3r_2+s_2} = \left(\frac{1}{\gamma^3}\right)^{m+1} y_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(5)}_{s_2-4-3\lfloor \frac{s_2-1}{3} \rfloor} + \zeta\left(4p + r_2 + 1 + \lfloor \frac{s_2-1}{3} \rfloor\right)}{\tilde{v}^{(5)}_{s_2-4-3\lfloor \frac{s_2-1}{3} \rfloor} + \zeta\left(4p + r_2 + \lfloor \frac{s_2-1}{3} \rfloor\right)}, \tag{82}$$

$$z_{12m+3r_2+s_1} = \gamma^{m+1} z_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(5)}_{s_1-3} + \zeta\left(4p + r_2 - 2\right)}{\tilde{v}^{(5)}_{s_1-3} + \zeta\left(4p + r_2 + 1\right)},$$

if $\eta = 1$, where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case 16: Let $\gamma = \zeta = 0$ and $\alpha\beta\theta\eta \neq 0$. In this case, we get the following system

$$x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{y_{n-1}z_{n-2}\left(\alpha + \beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}\right)} \quad y_n = \frac{1}{\theta z_{n-1}x_{n-2}y_{n-3}}, \quad z_n = \frac{x_{n-4}y_{n-5}z_{n-6}}{\eta x_{n-1}y_{n-2}}, \quad n \in \mathbb{N}_0. \tag{83}$$

By interchanging y_n, z_n, x_n, α, β, η and θ instead of x_n, y_n, z_n, η, ζ, γ and β in system (73), we obtain system (83). So, the solutions in (81) and (82) turn into the following formulas

$$x_{12m+3r_2+s_1} = \eta^{m+1} x_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r_2-2}\left((1-\alpha)v^{(5)}_{s_1-3} - \beta\right)}{\beta + \alpha^{4p+r_2+1}\left((1-\alpha)v^{(5)}_{s_1-3} - \beta\right)},$$

$$y_{12m+3r_1+s_1} = \eta^{m+1} y_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r_1+\lfloor \frac{s_1-2}{3} \rfloor}\left((1-\alpha)v^{(5)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} - \beta\right)}{\beta + \alpha^{4p+r_1-1+\lfloor \frac{s_1-2}{3} \rfloor}\left((1-\alpha)v^{(5)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} - \beta\right)},$$

$$z_{12m+3r_2+s_2} = \left(\frac{1}{\eta^3}\right)^{m+1} z_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r_2+1+\lfloor \frac{s_2-1}{3} \rfloor}\left((1-\alpha)v^{(5)}_{s_2-4-3\lfloor \frac{s_2-1}{3} \rfloor} - \beta\right)}{\beta + \alpha^{4p+r_2+\lfloor \frac{s_2-1}{3} \rfloor}\left((1-\alpha)v^{(5)}_{s_2-4-3\lfloor \frac{s_2-1}{3} \rfloor} - \beta\right)},$$

if $\alpha \neq 1$, and

$$x_{12m+3r_2+s_1} = \eta^{m+1} x_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{v^{(5)}_{s_1-3} + \beta\left(4p + r_2 - 2\right)}{v^{(5)}_{s_1-3} + \beta\left(4p + r_2 + 1\right)},$$

$$y_{12m+3r_1+s_1} = \eta^{m+1} y_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{v^{(5)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} + \beta\left(4p + r_1 + \lfloor \frac{s_1-2}{3} \rfloor\right)}{v^{(5)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} + \beta\left(4p + r_1 - 1 + \lfloor \frac{s_1-2}{3} \rfloor\right)},$$

$$z_{12m+3r_2+s_2} = \left(\frac{1}{\eta^3}\right)^{m+1} z_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{v^{(5)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} + \beta\left(4p + r_2 + 1 + \lfloor\frac{s_2-1}{3}\rfloor\right)}{v^{(5)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} + \beta\left(4p + r_2 + \lfloor\frac{s_2-1}{3}\rfloor\right)},$$

if $\alpha = 1$, where $m \in \mathbb{N}_0$, $r_1 \in \{3, 4, 5, 6\}$, $r_2 \in \{2, 3, 4, 5\}$, $s_1 \in \{0, 1, 2\}$ and $s_2 \in \{2, 3, 4\}$.

Case 17: $\beta = \eta = 0$ and $\alpha\gamma\theta\zeta \neq 0$. In this case, we get the following system

$$x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{\alpha y_{n-1}z_{n-2}}, \quad y_n = \frac{z_{n-4}x_{n-5}y_{n-6}}{z_{n-1}x_{n-2}\left(\gamma + \theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}\right)}, \quad z_n = \frac{1}{\zeta x_{n-1}y_{n-2}z_{n-3}}, \quad n \in \mathbb{N}_0. \tag{84}$$

By interchanging z_n, x_n, y_n, γ, θ, α and ζ instead of x_n, y_n, z_n, η, ζ, γ and β in system (73), we obtain system (84). So, the solutions in (81) and (82) turn into the following formulas

$$x_{12m+3r_2+s_2} = \left(\frac{1}{\alpha^3}\right)^{m+1} x_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\gamma)\hat{\vartheta}^{(5)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} - \theta\right)}{\theta + \gamma^{4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\gamma)\hat{\vartheta}^{(5)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} - \theta\right)},$$

$$y_{12m+3r_2+s_1} = \alpha^{m+1} y_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r_2-2}\left((1-\gamma)\hat{\vartheta}^{(5)}_{s_1-3} - \theta\right)}{\theta + \gamma^{4p+r_2+1}\left((1-\gamma)\hat{\vartheta}^{(5)}_{s_1-3} - \theta\right)},$$

$$z_{12m+3r_1+s_1} = \alpha^{m+1} z_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{\vartheta}^{(5)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \theta\right)}{\theta + \gamma^{4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{\vartheta}^{(5)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \theta\right)},$$

if $\gamma \neq 1$, and

$$x_{12m+3r_2+s_2} = \left(\frac{1}{\alpha^3}\right)^{m+1} x_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\hat{\vartheta}^{(5)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} + \theta\left(4p + r_2 + 1 + \lfloor\frac{s_2-1}{3}\rfloor\right)}{\hat{\vartheta}^{(5)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} + \theta\left(4p + r_2 + \lfloor\frac{s_2-1}{3}\rfloor\right)},$$

$$y_{12m+3r_2+s_1} = \alpha^{m+1} y_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{\hat{\vartheta}^{(5)}_{s_1-3} + \theta\left(4p + r_2 - 2\right)}{\hat{\vartheta}^{(5)}_{s_1-3} + \theta\left(4p + r_2 + 1\right)},$$

$$z_{12m+3r_1+s_1} = \alpha^{m+1} z_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\hat{\vartheta}^{(5)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} + \theta\left(4p + r_1 + \lfloor\frac{s_1-2}{3}\rfloor\right)}{\hat{\vartheta}^{(5)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} + \theta\left(4p + r_1 - 1\lfloor\frac{s_1-2}{3}\rfloor\right)},$$

if $\gamma = 1$, where $m \in \mathbb{N}_0$, $r_1 \in \{3, 4, 5, 6\}$, $r_2 \in \{2, 3, 4, 5\}$, $s_1 \in \{0, 1, 2\}$ and $s_2 \in \{2, 3, 4\}$.

Case 18: Let $\alpha = \zeta = 0$ and $\beta\gamma\theta\eta \neq 0$. In this case, system (12) turns into the following system

$$x_n = \frac{1}{\beta y_{n-1}z_{n-2}x_{n-3}}, \quad y_n = \frac{z_{n-4}x_{n-5}y_{n-6}}{z_{n-1}x_{n-2}\left(\gamma + \theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}\right)}, \quad z_n = \frac{x_{n-4}y_{n-5}z_{n-6}}{\eta x_{n-1}y_{n-2}}, \quad n \in \mathbb{N}_0. \tag{85}$$

Multiplying the first equation in system (85) by $y_{n-1}z_{n-2}x_{n-3}$ for all $n \in \mathbb{N}_0$, the second by $z_{n-1}x_{n-2}y_{n-3}$ for all $n \in \mathbb{N}_0$ and the third by $x_{n-1}y_{n-2}z_{n-3}$ for all $n \in \mathbb{N}_0$, it follows that

$$x_n y_{n-1}z_{n-2}x_{n-3} = \frac{1}{\beta}, \quad y_n z_{n-1}x_{n-2}y_{n-3} = \frac{y_{n-3}z_{n-4}x_{n-5}y_{n-6}}{\gamma + \theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}}, \quad z_n x_{n-1}y_{n-2}z_{n-3} = \frac{z_{n-3}x_{n-4}y_{n-5}z_{n-6}}{\eta}, \quad n \in \mathbb{N}_0.$$

(86)

By using the change of variables

$$\begin{cases} x_n y_{n-1} z_{n-2} x_{n-3} = & v_n^{(6)}, \quad n \in \mathbb{N}_0, \\ y_n z_{n-1} x_{n-2} y_{n-3} = & \frac{1}{\hat{v}_n^{(6)}}, \quad n \geq -3, \\ z_n x_{n-1} y_{n-2} z_{n-3} = & \tilde{v}_n^{(6)}, \quad n \geq -3, \end{cases} \tag{87}$$

system (86) becomes

$$v_n^{(6)} = \frac{1}{\beta}, \quad \hat{v}_n^{(6)} = \gamma \hat{v}_{n-3}^{(6)} + \theta, \quad \tilde{v}_n^{(6)} = \frac{1}{\eta} \tilde{v}_{n-3}^{(6)}, \quad n \in \mathbb{N}_0. \tag{88}$$

From(4), the solution the second equation in (88) and from (2), the solution of the third equation in (88) is given respectively

$$\begin{cases} \hat{v}_{3m+i}^{(6)} = & \begin{cases} \frac{\theta + \gamma^{m+1}\left((1-\gamma)\hat{v}_{i-3}^{(6)} - \theta\right)}{1-\gamma}, & \gamma \neq 1, \\ \hat{v}_{i-3}^{(6)} + \theta\,(m+1), & \gamma = 1, \end{cases} \\ \tilde{v}_{3m+i}^{(6)} = & \left(\frac{1}{\eta}\right)^{m+1} \tilde{v}_{i-3}^{(6)}, \end{cases} \tag{89}$$

where $m \in \mathbb{N}_0$ and $i \in \{0, 1, 2\}$.
From (87), we gain

$$\begin{cases} x_n = & \frac{v_n^{(6)} \hat{v}_{n-1}^{(6)} \tilde{v}_{n-8}^{(6)}}{\hat{v}_{n-4}^{(6)} \tilde{v}_{n-5}^{(6)} v_{n-9}^{(6)}} x_{n-12}, \quad n \geq 9, \\ y_n = & \frac{\tilde{v}_{n-4}^{(6)} v_{n-8}^{(6)} \hat{v}_{n-9}^{(6)}}{\hat{v}_n^{(6)} \tilde{v}_{n-1}^{(6)} v_{n-5}^{(6)}} y_{n-12}, \quad n \geq 8, \\ z_n = & \frac{\tilde{v}_n^{(6)} v_{n-4}^{(6)} \hat{v}_{n-5}^{(6)}}{v_{n-1}^{(6)} \hat{v}_{n-8}^{(6)} \tilde{v}_{n-9}^{(6)}} z_{n-12}, \quad n \geq 6, \end{cases} \tag{90}$$

and eventually

$$\begin{cases} x_{12m+j_1} = & \frac{v_{12m+j_1}^{(6)} \hat{v}_{12m+j_1-1}^{(6)} \tilde{v}_{12m+j_1-8}^{(6)}}{\hat{v}_{12m+j_1-4}^{(6)} \tilde{v}_{12m+j_1-5}^{(6)} v_{12m+j_1-9}^{(6)}} x_{12(m-1)+j_1}, \\ y_{12m+j_2} = & \frac{\tilde{v}_{12m+j_2-4}^{(6)} v_{12m+j_2-8}^{(6)} \hat{v}_{12m+j_2-9}^{(6)}}{\hat{v}_{12m+j_2}^{(6)} \tilde{v}_{12m+j_2-1}^{(6)} v_{12m+j_2-5}^{(6)}} y_{12(m-1)+j_2}, \\ z_{12m+j_3} = & \frac{\tilde{v}_{12m+j_3}^{(6)} v_{12m+j_3-4}^{(6)} \hat{v}_{12m+j_3-5}^{(6)}}{v_{12m+j_3-1}^{(6)} \hat{v}_{12m+j_3-8}^{(6)} \tilde{v}_{12m+j_3-9}^{(6)}} z_{12(m-1)+j_3}, \end{cases} \tag{91}$$

where $m \in \mathbb{N}_0$, $j_1 = \overline{9, 20}$, $j_2 = \overline{8, 19}$ and $j_3 = \overline{6, 17}$.
From (91), we attain

$$\begin{aligned} x_{12m+3r_1+s_1} =& x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{v_{12p+3r_1+s_1}^{(6)} \hat{v}_{12p+3r_1+s_1-1}^{(6)} \tilde{v}_{12p+3r_1+s_1-8}^{(6)}}{\hat{v}_{12p+3r_1+s_1-4}^{(6)} \tilde{v}_{12p+3r_1+s_1-5}^{(6)} v_{12p+3r_1+s_1-9}^{(6)}}, \\ y_{12m+3r_2+s_2} =& y_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\tilde{v}_{12p+3r_2+s_2-4}^{(6)} v_{12p+3r_2+s_2-8}^{(6)} \hat{v}_{12p+3r_2+s_2-9}^{(6)}}{\hat{v}_{12p+3r_2+s_2}^{(6)} \tilde{v}_{12p+3r_2+s_2-1}^{(6)} v_{12p+3r_2+s_2-5}^{(6)}}, \\ z_{12m+3r_2+s_1} =& z_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{\tilde{v}_{12p+3r_2+s_1}^{(6)} v_{12p+3r_2+s_1-4}^{(6)} \hat{v}_{12p+3r_2+s_1-5}^{(6)}}{v_{12p+3r_2+s_1-1}^{(6)} \hat{v}_{12p+3r_2+s_1-8}^{(6)} \tilde{v}_{12p+3r_2+s_1-9}^{(6)}}, \end{aligned} \tag{92}$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.
From (89) and (92), we obtain

$$\begin{aligned}
x_{12m+3r_1+s_1} &= \eta^{m+1} x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r_1+1+\lfloor \frac{s_1-1}{3} \rfloor} \left((1-\gamma)\hat{\vartheta}^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} - \theta \right)}{\theta + \gamma^{4p+r_1+\lfloor \frac{s_1-1}{3} \rfloor} \left((1-\gamma)\hat{\vartheta}^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} - \theta \right)}, \\
y_{12m+3r_2+s_2} &= \eta^{m+1} y_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r_2-1+\lfloor \frac{s_2-3}{3} \rfloor} \left((1-\gamma)\hat{\vartheta}^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} - \theta \right)}{\theta + \gamma^{4p+r_2+2+\lfloor \frac{s_2-3}{3} \rfloor} \left((1-\gamma)\hat{\vartheta}^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} - \theta \right)}, \qquad (93) \\
z_{12m+3r_2+s_1} &= \left(\frac{1}{\eta^3} \right)^{m+1} z_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r_2+\lfloor \frac{s_1-2}{3} \rfloor} \left((1-\gamma)\hat{\vartheta}^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} - \theta \right)}{\theta + \gamma^{4p+r_2-1+\lfloor \frac{s_1-2}{3} \rfloor} \left((1-\gamma)\hat{\vartheta}^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} - \theta \right)},
\end{aligned}$$

if $\gamma \neq 1$, and

$$\begin{aligned}
x_{12m+3r_1+s_1} &= \eta^{m+1} x_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\hat{\vartheta}^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} + \theta \left(4p + r_1 + 1 + \lfloor \frac{s_1-1}{3} \rfloor \right)}{\hat{\vartheta}^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} + \theta \left(4p + r_1 + \lfloor \frac{s_1-1}{3} \rfloor \right)}, \\
y_{12m+3r_2+s_2} &= \eta^{m+1} y_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\hat{\vartheta}^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} + \theta \left(4p + r_2 - 1 + \lfloor \frac{s_2-3}{3} \rfloor \right)}{\hat{\vartheta}^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} + \theta \left(4p + r_2 + 2 + \lfloor \frac{s_2-3}{3} \rfloor \right)}, \qquad (94) \\
z_{12m+3r_2+s_1} &= \left(\frac{1}{\eta^3} \right)^{m+1} z_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{\hat{\vartheta}^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} + \theta \left(4p + r_2 + \lfloor \frac{s_1-2}{3} \rfloor \right)}{\hat{\vartheta}^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} + \theta \left(4p + r_2 - 1 + \lfloor \frac{s_1-2}{3} \rfloor \right)},
\end{aligned}$$

if $\gamma = 1$, where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case 19: Let $\beta = \gamma = 0$ and $\alpha\theta\eta\zeta \neq 0$. In this case, we get the following system

$$x_n = \frac{y_{n-4} z_{n-5} x_{n-6}}{\alpha y_{n-1} z_{n-2}}, \quad y_n = \frac{1}{\theta z_{n-1} x_{n-2} y_{n-3}}, \quad z_n = \frac{x_{n-4} y_{n-5} z_{n-6}}{x_{n-1} y_{n-2} \left(\eta + \zeta z_{n-3} x_{n-4} y_{n-5} z_{n-6} \right)}, \quad n \in \mathbb{N}_0. \qquad (95)$$

By interchanging y_n, z_n, x_n, α, θ, η and ζ instead of x_n, y_n, z_n, η, β, γ and θ in system (85), we obtain system (95). So, the solutions in (93) and (94) turn into the following formulas

$$\begin{aligned}
x_{12m+3r_2+s_1} &= \left(\frac{1}{\alpha^3} \right)^{m+1} x_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r_2+\lfloor \frac{s_1-2}{3} \rfloor} \left((1-\eta)\tilde{\vartheta}^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} - \zeta \right)}{\zeta + \eta^{4p+r_2-1+\lfloor \frac{s_1-2}{3} \rfloor} \left((1-\eta)\tilde{\vartheta}^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} - \zeta \right)}, \\
y_{12m+3r_1+s_1} &= \alpha^{m+1} y_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r_1+1+\lfloor \frac{s_1-1}{3} \rfloor} \left((1-\eta)\tilde{\vartheta}^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} - \zeta \right)}{\zeta + \eta^{4p+r_1+\lfloor \frac{s_1-1}{3} \rfloor} \left((1-\eta)\tilde{\vartheta}^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} - \zeta \right)},
\end{aligned}$$

$$z_{12m+3r_2+s_2} = \alpha^{m+1} z_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r_2-1+\lfloor \frac{s_2-3}{3} \rfloor}\left((1-\eta)\tilde{v}^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} - \zeta\right)}{\zeta + \eta^{4p+r_2+2+\lfloor \frac{s_2-3}{3} \rfloor}\left((1-\eta)\tilde{v}^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} - \zeta\right)},$$

if $\eta \neq 1$, and

$$x_{12m+3r_2+s_1} = \left(\frac{1}{\alpha^3}\right)^{m+1} x_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} + \zeta\left(4p + r_2 + \lfloor \frac{s_1-2}{3} \rfloor\right)}{\tilde{v}^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} + \zeta\left(4p + r_2 - 1 + \lfloor \frac{s_1-2}{3} \rfloor\right)},$$

$$y_{12m+3r_1+s_1} = \alpha^{m+1} y_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} + \zeta\left(4p + r_1 + 1 + \lfloor \frac{s_1-1}{3} \rfloor\right)}{\tilde{v}^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} + \zeta\left(4p + r_1 + \lfloor \frac{s_1-1}{3} \rfloor\right)},$$

$$z_{12m+3r_2+s_2} = \alpha^{m+1} z_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} + \zeta\left(4p + r_2 - 1 + \lfloor \frac{s_2-3}{3} \rfloor\right)}{\tilde{v}^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} + \zeta\left(4p + r_2 + 2 + \lfloor \frac{s_2-3}{3} \rfloor\right)},$$

if $\eta = 1$, where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case 20: Let $\theta = \eta = 0$ and $\alpha\beta\gamma\zeta \neq 0$. In this case, we get the following system

$$x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{y_{n-1}z_{n-2}\left(\alpha + \beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}\right)}, \quad y_n = \frac{z_{n-4}x_{n-5}y_{n-6}}{\gamma z_{n-1}x_{n-2}}, \quad z_n = \frac{1}{\zeta x_{n-1}y_{n-2}z_{n-3}}, \quad n \in \mathbb{N}_0. \tag{96}$$

By interchanging z_n, x_n, y_n, γ, ζ, α and β instead of x_n, y_n, z_n, η, β, γ and θ in system (85), we obtain system (96). So, the solutions in (93) and (94) turn into the following formulas

$$x_{12m+3r_2+s_2} = \gamma^{m+1} x_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r_2-1+\lfloor \frac{s_2-3}{3} \rfloor}\left((1-\alpha)v^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} - \beta\right)}{\beta + \alpha^{4p+r_2+2+\lfloor \frac{s_2-3}{3} \rfloor}\left((1-\alpha)v^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} - \beta\right)},$$

$$y_{12m+3r_2+s_1} = \left(\frac{1}{\gamma^3}\right)^{m+1} y_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r_2+\lfloor \frac{s_1-2}{3} \rfloor}\left((1-\alpha)v^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} - \beta\right)}{\beta + \alpha^{4p+r_2-1+\lfloor \frac{s_1-2}{3} \rfloor}\left((1-\alpha)v^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} - \beta\right)},$$

$$z_{12m+3r_1+s_1} = \gamma^{m+1} z_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r_1+1+\lfloor \frac{s_1-1}{3} \rfloor}\left((1-\alpha)v^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} - \beta\right)}{\beta + \alpha^{4p+r_1+\lfloor \frac{s_1-1}{3} \rfloor}\left((1-\alpha)v^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} - \beta\right)},$$

if $\alpha \neq 1$, and

$$x_{12m+3r_2+s_2} = \gamma^{m+1} x_{3r_2+s_2-12} \prod_{p=0}^{m} \frac{v^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} + \beta\left(4p + r_2 - 1 + \lfloor \frac{s_2-3}{3} \rfloor\right)}{v^{(6)}_{s_2-6-3\lfloor \frac{s_2-3}{3} \rfloor} + \beta\left(4p + r_2 + 2 + \lfloor \frac{s_2-3}{3} \rfloor\right)},$$

$$y_{12m+3r_2+s_1} = \left(\frac{1}{\gamma^3}\right)^{m+1} y_{3r_2+s_1-12} \prod_{p=0}^{m} \frac{v^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} + \beta\left(4p + r_2 + \lfloor \frac{s_1-2}{3} \rfloor\right)}{v^{(6)}_{s_1-5-3\lfloor \frac{s_1-2}{3} \rfloor} + \beta\left(4p + r_2 - 1 + \lfloor \frac{s_1-2}{3} \rfloor\right)},$$

$$z_{12m+3r_1+s_1} = \gamma^{m+1} z_{3r_1+s_1-12} \prod_{p=0}^{m} \frac{v^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} + \beta\left(4p + r_1 + 1 + \lfloor \frac{s_1-1}{3} \rfloor\right)}{v^{(6)}_{s_1-4-3\lfloor \frac{s_1-1}{3} \rfloor} + \beta\left(4p + r_1 + \lfloor \frac{s_1-1}{3} \rfloor\right)},$$

f $\alpha = 1$, where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case 21: Let $\alpha = 0$ and $\beta\gamma\theta\eta\zeta \neq 0$. In this case, system (12) transforms into the following system

$$x_n = \frac{1}{\beta y_{n-1} z_{n-2} x_{n-3}}, \quad y_n = \frac{z_{n-4} x_{n-5} y_{n-6}}{z_{n-1} x_{n-2} \left(\gamma + \theta y_{n-3} z_{n-4} x_{n-5} y_{n-6}\right)}, \quad z_n = \frac{x_{n-4} y_{n-5} z_{n-6}}{x_{n-1} y_{n-2} \left(\eta + \zeta z_{n-3} x_{n-4} y_{n-5} z_{n-6}\right)}, \tag{97}$$

for $n \in \mathbb{N}_0$.
Multiplying the first equation in system (97) by $y_{n-1} z_{n-2} x_{n-3}$ for all $n \in \mathbb{N}_0$, the second by $z_{n-1} x_{n-2} y_{n-3}$ for all $n \in \mathbb{N}_0$ and the third by $x_{n-1} y_{n-2} z_{n-3}$ for all $n \in \mathbb{N}_0$, it follows that

$$x_n y_{n-1} z_{n-2} x_{n-3} = \frac{1}{\beta}, \quad y_n z_{n-1} x_{n-2} y_{n-3} = \frac{y_{n-3} z_{n-4} x_{n-5} y_{n-6}}{\gamma + \theta y_{n-3} z_{n-4} x_{n-5} y_{n-6}}, \quad z_n x_{n-1} y_{n-2} z_{n-3} = \frac{z_{n-3} x_{n-4} y_{n-5} z_{n-6}}{\eta + \zeta z_{n-3} x_{n-4} y_{n-5} z_{n-6}}, \quad n \in \mathbb{N}_0. \tag{98}$$

By employing the change of variables

$$\begin{cases} x_n y_{n-1} z_{n-2} x_{n-3} = v_n^{(7)}, & n \in \mathbb{N}_0, \\ y_n z_{n-1} x_{n-2} y_{n-3} = \frac{1}{\hat{v}_n^{(7)}}, & n \geq -3, \\ z_n x_{n-1} y_{n-2} z_{n-3} = \frac{1}{\tilde{v}_n^{(7)}}, & n \geq -3, \end{cases} \tag{99}$$

system (98) becomes

$$v_n^{(7)} = \frac{1}{\beta}, \quad \hat{v}_n^{(7)} = \gamma \hat{v}_{n-3}^{(7)} + \theta, \quad \tilde{v}_n^{(7)} = \eta \tilde{v}_{n-3}^{(7)} + \zeta, \quad n \in \mathbb{N}_0. \tag{100}$$

From (4), the solutions of the second and the third equations in (100) is given respectively

$$\hat{v}_{3m+i}^{(7)} = \begin{cases} \frac{\theta + \gamma^{m+1}\left((1-\gamma)\hat{v}_{i-3}^{(7)} - \theta\right)}{1-\gamma}, & \gamma \neq 1, \\ \hat{v}_{i-3}^{(7)} + \theta\,(m+1), & \gamma = 1, \end{cases} \quad \tilde{v}_{3m+i}^{(7)} = \begin{cases} \frac{\zeta + \eta^{m+1}\left((1-\eta)\tilde{v}_{i-3}^{(7)} - \zeta\right)}{1-\eta}, & \eta \neq 1, \\ \tilde{v}_{i-3}^{(7)} + \zeta\,(m+1), & \eta = 1, \end{cases} \tag{101}$$

where $m \in \mathbb{N}_0$ and $i \in \{0,1,2\}$.
From (99), we gain

$$\begin{cases} x_n = \frac{v_n^{(7)} \hat{v}_{n-1}^{(7)} \tilde{v}_{n-5}^{(7)}}{\hat{v}_{n-4}^{(7)} \tilde{v}_{n-8}^{(7)} v_{n-9}^{(7)}} x_{n-12}, & n \geq 9, \\ y_n = \frac{\tilde{v}_{n-1}^{(7)} v_{n-8}^{(7)} \hat{v}_{n-9}^{(7)}}{\hat{v}_n^{(7)} \tilde{v}_{n-4}^{(7)} v_{n-5}^{(7)}} y_{n-12}, & n \geq 8, \\ z_n = \frac{v_{n-4}^{(7)} \hat{v}_{n-5}^{(7)} \tilde{v}_{n-9}^{(7)}}{\tilde{v}_n^{(7)} v_{n-1}^{(7)} \hat{v}_{n-8}^{(7)}} z_{n-12}, & n \geq 6, \end{cases} \tag{102}$$

and eventually

$$\begin{cases} x_{12m+j_1} = \frac{v_{12m+j_1}^{(7)} \hat{v}_{12m+j_1-1}^{(7)} \tilde{v}_{12m+j_1-5}^{(7)}}{\hat{v}_{12m+j_1-4}^{(7)} \tilde{v}_{12m+j_1-8}^{(7)} v_{12m+j_1-9}^{(7)}} x_{12(m-1)+j_1}, \\ y_{12m+j_2} = \frac{\tilde{v}_{12m+j_2-1}^{(7)} v_{12m+j_2-8}^{(7)} \hat{v}_{12m+j_2-9}^{(7)}}{\hat{v}_{12m+j_2}^{(7)} \tilde{v}_{12m+j_2-4}^{(7)} v_{12m+j_2-5}^{(7)}} y_{12(m-1)+j_2}, \\ z_{12m+j_3} = \frac{v_{12m+j_3-4}^{(7)} \hat{v}_{12m+j_3-5}^{(7)} \tilde{v}_{12m+j_3-9}^{(7)}}{\tilde{v}_{12m+j_3}^{(7)} v_{12m+j_3-1}^{(7)} \hat{v}_{12m+j_3-8}^{(7)}} z_{12(m-1)+j_3}, \end{cases} \tag{103}$$

where $m \in \mathbb{N}_0$, $j_1 = \overline{9,20}$, $j_2 = \overline{8,19}$ and $j_3 = \overline{6,17}$.
From (103), we attain

$$\begin{aligned}
x_{12m+3r_1+s_1} &= x_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{v^{(7)}_{12p+3r_1+s_1}\hat{v}^{(7)}_{12p+3r_1+s_1-1}\tilde{v}^{(7)}_{12p+3r_1+s_1-5}}{\hat{v}^{(7)}_{12p+3r_1+s_1-4}\tilde{v}^{(7)}_{12p+3r_1+s_1-8}v^{(7)}_{12p+3r_1+s_1-9}},\\
y_{12m+3r_2+s_2} &= y_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(7)}_{12p+3r_2+s_2-1}v^{(7)}_{12p+3r_2+s_2-8}\hat{v}^{(7)}_{12p+3r_2+s_2-9}}{\hat{v}^{(7)}_{12p+3r_2+s_2}\tilde{v}^{(7)}_{12p+3r_2+s_2-4}v^{(7)}_{12p+3r_2+s_2-5}},\\
z_{12m+3r_2+s_1} &= z_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{v^{(7)}_{12p+3r_2+s_1-4}\hat{v}^{(7)}_{12p+3r_2+s_1-5}\tilde{v}^{(7)}_{12p+3r_2+s_1-9}}{\tilde{v}^{(7)}_{12p+3r_2+s_1}v^{(7)}_{12p+3r_2+s_1-1}\hat{v}^{(7)}_{12p+3r_2+s_1-8}},
\end{aligned} \tag{104}$$

where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.
From (101) and (104), we obtain

$$\begin{aligned}
x_{12m+3r_1+s_1} &= x_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{\theta+\gamma^{4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}-\theta\right)}{\theta+\gamma^{4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}-\theta\right)}\frac{\zeta+\eta^{4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\zeta\right)}{\zeta+\eta^{4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\zeta\right)},\\
y_{12m+3r_2+s_2} &= y_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\zeta+\eta^{4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}-\zeta\right)}{\theta+\gamma^{4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}-\theta\right)}\frac{\theta+\gamma^{4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}-\theta\right)}{\zeta+\eta^{4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}-\zeta\right)},\\
z_{12m+3r_2+s_1} &= z_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{\theta+\gamma^{4p+r_2+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\theta\right)}{\zeta+\eta^{4p+r_2+1}\left((1-\eta)\tilde{v}^{(7)}_{s_1-3}-\zeta\right)}\frac{\zeta+\eta^{4p+r_2-2}\left((1-\eta)\tilde{v}^{(7)}_{s_1-3}-\zeta\right)}{\theta+\gamma^{4p+r_2-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\theta\right)},
\end{aligned} \tag{105}$$

if $\gamma \neq 1$, $\eta \neq 1$, and

$$\begin{aligned}
x_{12m+3r_1+s_1} &= x_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{\hat{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}+\theta\left(4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor\right)}{\hat{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}+\theta\left(4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor\right)}\frac{\zeta+\eta^{4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\zeta\right)}{\zeta+\eta^{4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\zeta\right)},\\
y_{12m+3r_2+s_2} &= y_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\zeta+\eta^{4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}-\zeta\right)}{\hat{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}+\theta\left(4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor\right)}\frac{\hat{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}+\theta\left(4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor\right)}{\zeta+\eta^{4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}-\zeta\right)},\\
z_{12m+3r_2+s_1} &= z_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\theta\left(4p+r_2+\lfloor\frac{s_1-2}{3}\rfloor\right)}{\zeta+\eta^{4p+r_2+1}\left((1-\eta)\tilde{v}^{(7)}_{s_1-3}-\zeta\right)}\frac{\zeta+\eta^{4p+r_2-2}\left((1-\eta)\tilde{v}^{(7)}_{s_1-3}-\zeta\right)}{\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\theta\left(4p+r_2-1+\lfloor\frac{s_1-2}{3}\rfloor\right)},
\end{aligned} \tag{106}$$

if $\gamma = 1$, $\eta \neq 1$, and

$$\begin{aligned}
x_{12m+3r_1+s_1} &= x_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{\theta+\gamma^{4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}-\theta\right)}{\theta+\gamma^{4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}-\theta\right)}\frac{\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\zeta\left(4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor\right)}{\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\zeta\left(4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor\right)},\\
y_{12m+3r_2+s_2} &= y_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}+\zeta\left(4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor\right)}{\theta+\gamma^{4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}-\theta\right)}\frac{\theta+\gamma^{4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}-\theta\right)}{\tilde{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}+\zeta\left(4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor\right)},\\
z_{12m+3r_2+s_1} &= z_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{\theta+\gamma^{4p+r_2+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\theta\right)}{\tilde{v}^{(7)}_{s_1-3}+\zeta\left(4p+r_2+1\right)}\frac{\tilde{v}^{(7)}_{s_1-3}+\zeta\left(4p+r_2-2\right)}{\theta+\gamma^{4p+r_2-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\theta\right)},
\end{aligned}\tag{107}$$

if $\gamma \neq 1$, $\eta = 1$, and

$$\begin{aligned}
x_{12m+3r_1+s_1} &= x_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{\hat{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}+\theta\left(4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor\right)}{\hat{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}+\theta\left(4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor\right)}\frac{\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\zeta\left(4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor\right)}{\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\zeta\left(4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor\right)},\\
y_{12m+3r_2+s_2} &= x_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}+\zeta\left(4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor\right)}{\hat{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}+\theta\left(4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor\right)}\frac{\hat{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}+\theta\left(4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor\right)}{\tilde{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}+\zeta\left(4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor\right)},\\
z_{12m+3r_2+s_1} &= z_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\theta\left(4p+r_2+\lfloor\frac{s_1-2}{3}\rfloor\right)}{\tilde{v}^{(7)}_{s_1-3}+\zeta\left(4p+r_2+1\right)}\frac{\tilde{v}^{(7)}_{s_1-3}+\zeta\left(4p+r_2-2\right)}{\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\theta\left(4p+r_2-1+\lfloor\frac{s_1-2}{3}\rfloor\right)},
\end{aligned}\tag{108}$$

if $\gamma = 1$, $\eta = 1$, where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case 22: Let $\gamma = 0$ and $\alpha\beta\theta\eta\zeta \neq 0$. In this case, we get the following system

$$x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{y_{n-1}z_{n-2}\left(\alpha+\beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}\right)},\quad y_n = \frac{1}{\theta z_{n-1}x_{n-2}y_{n-3}},\quad z_n = \frac{x_{n-4}y_{n-5}z_{n-6}}{x_{n-1}y_{n-2}\left(\eta+\zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}\right)},\tag{109}$$

for $n \in \mathbb{N}_0$.

By interchanging y_n, z_n, x_n, θ, η, ζ, α and β instead of x_n, y_n, z_n, β, γ, θ, η and ζ system (97) transforms into system (109). So, the solutions in (105)-(108) turn into the following formulas

$$\begin{aligned}
x_{12m+3r_2+s_1} &= x_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{\zeta+\eta^{4p+r_2+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\zeta\right)}{\beta+\alpha^{4p+r_2+1}\left((1-\alpha)v^{(7)}_{s_1-3}-\beta\right)}\frac{\beta+\alpha^{4p+r_2-2}\left((1-\alpha)v^{(7)}_{s_1-3}-\beta\right)}{\zeta+\eta^{4p+r_2-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\zeta\right)},\\
y_{12m+3r_1+s_1} &= y_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\beta\right)}{\beta+\alpha^{4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\beta\right)}\frac{\zeta+\eta^{4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}-\zeta\right)}{\zeta+\eta^{4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}-\zeta\right)},
\end{aligned}$$

$$z_{12m+3r_2+s_2} = z_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}-\beta\right)}{\zeta+\eta^{4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}-\zeta\right)}\frac{\zeta+\eta^{4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}-\zeta\right)}{\beta+\alpha^{4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}-\beta\right)},$$

if $\alpha \neq 1$, $\eta \neq 1$, and

$$x_{12m+3r_2+s_1} = x_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{\zeta+\eta^{4p+r_2+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\zeta\right)}{v^{(7)}_{s_1-3}+\beta(4p+r_2+1)}\frac{v^{(7)}_{s_1-3}+\beta(4p+r_2-2)}{\zeta+\eta^{4p+r_2-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\zeta\right)},$$

$$y_{12m+3r_1+s_1} = y_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\beta\left(4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor\right)}{v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\beta\left(4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor\right)}\frac{\zeta+\eta^{4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}-\zeta\right)}{\zeta+\eta^{4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}-\zeta\right)},$$

$$z_{12m+3r_2+s_2} = z_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{v^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}+\beta\left(4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor\right)}{\zeta+\eta^{4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}-\zeta\right)}\frac{\zeta+\eta^{4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\eta)\tilde{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}-\zeta\right)}{v^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}+\beta\left(4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor\right)},$$

if $\alpha = 1$, $\eta \neq 1$, and

$$x_{12m+3r_2+s_1} = x_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\zeta\left(4p+r_2+\lfloor\frac{s_1-2}{3}\rfloor\right)}{\beta+\alpha^{4p+r_2+1}\left((1-\alpha)v^{(7)}_{s_1-3}-\beta\right)}\frac{\beta+\alpha^{4p+r_2-2}\left((1-\alpha)v^{(7)}_{s_1-3}-\beta\right)}{\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\zeta\left(4p+r_2-1+\lfloor\frac{s_1-2}{3}\rfloor\right)},$$

$$y_{12m+3r_1+s_1} = y_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\beta\right)}{\beta+\alpha^{4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}-\beta\right)}\frac{\tilde{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}+\zeta\left(4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor\right)}{\tilde{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}+\zeta\left(4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor\right)},$$

$$z_{12m+3r_2+s_2} = z_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}-\beta\right)}{\tilde{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}+\zeta\left(4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor\right)}\frac{\tilde{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}+\zeta\left(4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor\right)}{\beta+\alpha^{4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}-\beta\right)},$$

if $\alpha \neq 1$, $\eta = 1$, and

$$x_{12m+3r_2+s_1} = x_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\zeta\left(4p+r_2+\lfloor\frac{s_1-2}{3}\rfloor\right)}{v^{(7)}_{s_1-3}+\beta(4p+r_2+1)}\frac{v^{(7)}_{s_1-3}+\beta(4p+r_2-2)}{\tilde{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\zeta\left(4p+r_2-1+\lfloor\frac{s_1-2}{3}\rfloor\right)},$$

$$y_{12m+3r_1+s_1} = y_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\beta\left(4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor\right)}{v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\beta\left(4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor\right)}\frac{\tilde{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}+\zeta\left(4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor\right)}{\tilde{v}^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}+\zeta\left(4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor\right)},$$

$$z_{12m+3r_2+s_2} = z_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{v^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}+\beta\left(4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor\right)}{\tilde{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}+\zeta\left(4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor\right)}\frac{\tilde{v}^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}+\zeta\left(4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor\right)}{v^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}+\beta\left(4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor\right)},$$

if $\alpha = 1$, $\eta = 1$, where $m \in \mathbb{N}_0$, $r_1 \in \{3,4,5,6\}$, $r_2 \in \{2,3,4,5\}$, $s_1 \in \{0,1,2\}$ and $s_2 \in \{2,3,4\}$.

Case: 23: Let $\eta = 0$ and $\alpha\beta\gamma\theta\zeta \neq 0$. In this case, we get the following system

$$x_n = \frac{y_{n-4}z_{n-5}x_{n-6}}{y_{n-1}z_{n-2}\left(\alpha + \beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}\right)}, \quad y_n = \frac{z_{n-4}x_{n-5}y_{n-6}}{z_{n-1}x_{n-2}\left(\gamma + \theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}\right)}, \quad z_n = \frac{1}{\zeta x_{n-1}y_{n-2}z_{n-3}}, \tag{110}$$

for $n \in \mathbb{N}_0$.
By interchanging z_n, x_n, y_n, ζ, α, β, γ and θ instead of x_n, y_n, z_n, β, γ, θ, η and ζ system (97) transforms into system (110). So, the solutions in (105)-(108) turn into the following formulas

$$x_{12m+3r_2+s_2} = x_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\theta + \gamma^{4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} - \theta\right)}{\beta + \alpha^{4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} - \beta\right)}\frac{\beta + \alpha^{4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} - \beta\right)}{\theta + \gamma^{4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} - \theta\right)},$$

$$y_{12m+3r_2+s_1} = y_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{\beta + \alpha^{4p+r_2+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \beta\right)}{\theta + \gamma^{4p+r_2+1}\left((1-\gamma)\hat{v}^{(7)}_{s_1-3} - \theta\right)}\frac{\theta + \gamma^{4p+r_2-2}\left((1-\gamma)\hat{v}^{(7)}_{s_1-3} - \theta\right)}{\beta + \alpha^{4p+r_2-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \beta\right)},$$

$$z_{12m+3r_1+s_1} = z_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{\beta + \alpha^{4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} - \beta\right)}{\beta + \alpha^{4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} - \beta\right)}\frac{\theta + \gamma^{4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \theta\right)}{\theta + \gamma^{4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \theta\right)},$$

if $\alpha \neq 1$, $\gamma \neq 1$, and

$$x_{12m+3r_2+s_2} = x_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\theta + \gamma^{4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} - \theta\right)}{v^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} + \beta\left(4p + r_2 + 2 + \lfloor\frac{s_2-3}{3}\rfloor\right)}\frac{v^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} + \beta\left(4p + r_2 - 1 + \lfloor\frac{s_2-3}{3}\rfloor\right)}{\theta + \gamma^{4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} - \theta\right)},$$

$$y_{12m+3r_2+s_1} = y_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} + \beta\left(4p + r_2 + \lfloor\frac{s_1-2}{3}\rfloor\right)}{\theta + \gamma^{4p+r_2+1}\left((1-\gamma)\hat{v}^{(7)}_{s_1-3} - \theta\right)}\frac{\theta + \gamma^{4p+r_2-2}\left((1-\gamma)\hat{v}^{(7)}_{s_1-3} - \theta\right)}{v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} + \beta\left(4p + r_2 - 1 + \lfloor\frac{s_1-2}{3}\rfloor\right)},$$

$$z_{12m+3r_1+s_1} = z_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{v^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} + \beta\left(4p + r_1 + 1 + \lfloor\frac{s_1-1}{3}\rfloor\right)}{v^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor} + \beta\left(4p + r_1 + \lfloor\frac{s_1-1}{3}\rfloor\right)}\frac{\theta + \gamma^{4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \theta\right)}{\theta + \gamma^{4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \theta\right)},$$

if $\alpha = 1$, $\gamma \neq 1$, and

$$x_{12m+3r_2+s_2} = x_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\hat{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} + \theta\left(4p + r_2 + 1 + \lfloor\frac{s_2-1}{3}\rfloor\right)}{\beta + \alpha^{4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} - \beta\right)}\frac{\beta + \alpha^{4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor} - \beta\right)}{\hat{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor} + \theta\left(4p + r_2 + \lfloor\frac{s_2-1}{3}\rfloor\right)},$$

$$y_{12m+3r_2+s_1} = y_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{\beta + \alpha^{4p+r_2+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \beta\right)}{\hat{v}^{(7)}_{s_1-3} + \theta\left(4p + r_2 + 1\right)}\frac{\hat{v}^{(7)}_{s_1-3} + \theta\left(4p + r_2 - 2\right))}{\beta + \alpha^{4p+r_2-1+\lfloor\frac{s_1-2}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor} - \beta\right)},$$

$$z_{12m+3r_1+s_1}=z_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}-\beta\right)}{\beta+\alpha^{4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor}\left((1-\alpha)v^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}-\beta\right)}\frac{\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\theta\left(4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor\right)}{\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\theta\left(4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor\right)},$$

if $\alpha\neq 1$, $\gamma=1$, and

$$x_{12m+3r_2+s_2}=x_{3r_2+s_2-12}\prod_{p=0}^{m}\frac{\hat{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}+\theta\left(4p+r_2+1+\lfloor\frac{s_2-1}{3}\rfloor\right)}{v^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}+\beta\left(4p+r_2+2+\lfloor\frac{s_2-3}{3}\rfloor\right)}\frac{v^{(7)}_{s_2-6-3\lfloor\frac{s_2-3}{3}\rfloor}+\beta\left(4p+r_2-1+\lfloor\frac{s_2-3}{3}\rfloor\right)}{\hat{v}^{(7)}_{s_2-4-3\lfloor\frac{s_2-1}{3}\rfloor}+\theta\left(4p+r_2+\lfloor\frac{s_2-1}{3}\rfloor\right)},$$

$$y_{12m+3r_2+s_1}=y_{3r_2+s_1-12}\prod_{p=0}^{m}\frac{v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\beta\left(4p+r_2+\lfloor\frac{s_1-2}{3}\rfloor\right)}{\hat{v}^{(7)}_{s_1-3}+\theta\left(4p+r_2+1\right)}\frac{\hat{v}^{(7)}_{s_1-3}+\theta\left(4p+r_2-2\right)}{v^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\beta\left(4p+r_2-1+\lfloor\frac{s_1-2}{3}\rfloor\right)},$$

$$z_{12m+3r_1+s_1}=z_{3r_1+s_1-12}\prod_{p=0}^{m}\frac{v^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}+\beta\left(4p+r_1+1+\lfloor\frac{s_1-1}{3}\rfloor\right)}{v^{(7)}_{s_1-4-3\lfloor\frac{s_1-1}{3}\rfloor}+\beta\left(4p+r_1+\lfloor\frac{s_1-1}{3}\rfloor\right)}\frac{\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\theta\left(4p+r_1+\lfloor\frac{s_1-2}{3}\rfloor\right)}{\hat{v}^{(7)}_{s_1-5-3\lfloor\frac{s_1-2}{3}\rfloor}+\theta\left(4p+r_1-1+\lfloor\frac{s_1-2}{3}\rfloor\right)},$$

if $\gamma=\eta=1$, where $m\in\mathbb{N}_0$, $r_1\in\{3,4,5,6\}$, $r_2\in\{2,3,4,5\}$, $s_1\in\{0,1,2\}$ and $s_2\in\{2,3,4\}$.

Case 24: Let $\beta=0$ and $\alpha\gamma\theta\eta\zeta\neq 0$. In this case, the system (12) transforms into the following system

$$x_n=\frac{y_{n-4}z_{n-5}x_{n-6}}{\alpha y_{n-1}z_{n-2}},\quad y_n=\frac{z_{n-4}x_{n-5}y_{n-6}}{z_{n-1}x_{n-2}\left(\gamma+\theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}\right)},\quad z_n=\frac{x_{n-4}y_{n-5}z_{n-6}}{x_{n-1}y_{n-2}\left(\eta+\zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}\right)},\tag{111}$$

for $n\in\mathbb{N}_0$. Multiplying the first equation in system (111) by $y_{n-1}z_{n-2}x_{n-3}$ for all $n\in\mathbb{N}_0$, the second by $z_{n-1}x_{n-2}y_{n-3}$ for all $n\in\mathbb{N}_0$ and the third by $x_{n-1}y_{n-2}z_{n-3}$ for all $n\in\mathbb{N}_0$, it follows that

$$\begin{cases}x_ny_{n-1}z_{n-2}x_{n-3}= & \frac{x_{n-3}y_{n-4}z_{n-5}x_{n-6}}{\alpha},\\ y_nz_{n-1}x_{n-2}y_{n-3}= & \frac{y_{n-3}z_{n-4}x_{n-5}y_{n-6}}{\gamma+\theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}},\\ z_nx_{n-1}y_{n-2}z_{n-3}= & \frac{z_{n-3}x_{n-4}y_{n-5}z_{n-6}}{\eta+\zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}},\end{cases}\quad n\in\mathbb{N}_0,\tag{112}$$

By using the change of variables

$$x_ny_{n-1}z_{n-2}x_{n-3}=v_n^{(8)},\quad y_nz_{n-1}x_{n-2}y_{n-3}=\frac{1}{\hat{v}_n^{(8)}},\quad z_nx_{n-1}y_{n-2}z_{n-3}=\frac{1}{\tilde{v}_n^{(8)}},\quad n\geq -3,\tag{113}$$

system (112) becomes

$$v_n^{(8)}=\frac{1}{\alpha}v^{(8)}_{n-3},\quad \hat{v}_n^{(8)}=\gamma\hat{v}^{(8)}_{n-3}+\theta,\quad \tilde{v}_n^{(8)}=\eta\tilde{v}^{(8)}_{n-3}+\zeta,\quad n\in\mathbb{N}_0.\tag{114}$$

From (2), the solution of the first equation in (114), from (4), the solutions of the second and the third equations in (114) is given respectively

$$\begin{cases}v^{(8)}_{3m+i}= & \left(\frac{1}{\alpha}\right)^{m+1}v^{(8)}_{i-3},\\ \hat{v}^{(8)}_{3m+i}= & \begin{cases}\frac{\theta+\gamma^{m+1}\left((1-\gamma)\hat{v}^{(8)}_{i-3}-\theta\right)}{1-\gamma}, & \gamma\neq 1,\\ \hat{v}^{(8)}_{i-3}+\theta\left(m+1\right), & \gamma=1,\end{cases}\\ \tilde{v}^{(8)}_{3m+i}= & \begin{cases}\frac{\zeta+\eta^{m+1}\left((1-\eta)\tilde{v}^{(8)}_{i-3}-\zeta\right)}{1-\eta}, & \eta\neq 1,\\ \tilde{v}^{(8)}_{i-3}+\zeta\left(m+1\right), & \eta=1,\end{cases}\end{cases}\tag{115}$$

where $m \in \mathbb{N}_0$ and $i \in \{0, 1, 2\}$.
From (113), we gain

$$\begin{cases} x_n = \dfrac{v_n^{(8)} \hat{v}_{n-1}^{(8)} \tilde{v}_{n-5}^{(8)}}{\hat{v}_{n-4}^{(8)} \tilde{v}_{n-8}^{(8)} v_{n-9}^{(8)}} x_{n-12}, \\ y_n = \dfrac{\tilde{v}_{n-1}^{(8)} v_{n-8}^{(8)} \hat{v}_{n-9}^{(8)}}{\hat{v}_n^{(8)} \tilde{v}_{n-4}^{(8)} v_{n-5}^{(8)}} y_{n-12}, \quad n \geq 6, \\ z_n = \dfrac{v_{n-4}^{(8)} \hat{v}_{n-5}^{(8)} \tilde{v}_{n-9}^{(8)}}{\tilde{v}_n^{(8)} v_{n-1}^{(8)} \hat{v}_{n-8}^{(8)}} z_{n-12}, \end{cases} \tag{116}$$

and eventually

$$\begin{cases} x_{12m+j} = \dfrac{v_{12m+j}^{(8)} \hat{v}_{12m+j-1}^{(8)} \tilde{v}_{12m+j-5}^{(8)}}{\hat{v}_{12m+j-4}^{(8)} \tilde{v}_{12m+j-8}^{(8)} v_{12m+j-9}^{(8)}} x_{12(m-1)+j}, \\ y_{12m+j} = \dfrac{\tilde{v}_{12m+j-1}^{(8)} v_{12m+j-8}^{(8)} \hat{v}_{12m+j-9}^{(8)}}{\hat{v}_{12m+j}^{(8)} \tilde{v}_{12m+j-4}^{(8)} v_{12m+j-5}^{(8)}} y_{12(m-1)+j}, \\ z_{12m+j} = \dfrac{v_{12m+j-4}^{(8)} \hat{v}_{12m+j-5}^{(8)} \tilde{v}_{12m+j-9}^{(8)}}{\tilde{v}_{12m+j}^{(8)} v_{12m+j-1}^{(8)} \hat{v}_{12m+j-8}^{(8)}} z_{12(m-1)+j}, \end{cases} \tag{117}$$

where $m \in \mathbb{N}_0$ and $j = \overline{6, 17}$.
From (117), we attain

$$\begin{aligned} x_{12m+3r+s} &= x_{3r+s-12} \prod_{p=0}^{m} \frac{v_{12p+3r+s}^{(8)} \hat{v}_{12p+3r+s-1}^{(8)} \tilde{v}_{12p+3r+s-5}^{(8)}}{\hat{v}_{12p+3r+s-4}^{(8)} \tilde{v}_{12p+3r+s-8}^{(8)} v_{12p+3r+s-9}^{(8)}}, \\ y_{12m+3r+s} &= y_{3r+s-12} \prod_{p=0}^{m} \frac{\tilde{v}_{12p+3r+s-1}^{(8)} v_{12p+3r+s-8}^{(8)} \hat{v}_{12p+3r+s-9}^{(8)}}{\hat{v}_{12p+3r+s}^{(8)} \tilde{v}_{12p+3r+s-4}^{(8)} v_{12p+3r+s-5}^{(8)}}, \\ z_{12m+3r+s} &= z_{3r+s-12} \prod_{p=0}^{m} \frac{v_{12p+3r+s-4}^{(8)} \hat{v}_{12p+3r+s-5}^{(8)} \tilde{v}_{12p+3r+s-9}^{(8)}}{\tilde{v}_{12p+3r+s}^{(8)} v_{12p+3r+s-1}^{(8)} \hat{v}_{12p+3r+s-8}^{(8)}}, \end{aligned} \tag{118}$$

where $m \in \mathbb{N}_0$, $r \in \{2, 3, 4, 5\}$ and $s \in \{0, 1, 2\}$.
From (115) and (118), we obtain

$$\begin{aligned} x_{12m+3r+s} &= \left(\frac{1}{\alpha^3}\right)^{m+1} x_{3r+s-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r+1+\lfloor \frac{s-1}{3} \rfloor} \left((1-\gamma)\hat{v}_{s-4-3\lfloor \frac{s-1}{3} \rfloor}^{(8)} - \theta\right)}{\theta + \gamma^{4p+r+\lfloor \frac{s-1}{3} \rfloor} \left((1-\gamma)\hat{v}_{s-4-3\lfloor \frac{s-1}{3} \rfloor}^{(8)} - \theta\right)} \frac{\zeta + \eta^{4p+r+\lfloor \frac{s-2}{3} \rfloor} \left((1-\eta)\tilde{v}_{s-5-3\lfloor \frac{s-2}{3} \rfloor}^{(8)} - \zeta\right)}{\zeta + \eta^{4p+r-1+\lfloor \frac{s-2}{3} \rfloor} \left((1-\eta)\tilde{v}_{s-5-3\lfloor \frac{s-2}{3} \rfloor}^{(8)} - \zeta\right)}, \\ y_{12m+3r+s} &= \alpha^{m+1} y_{3r+s-12} \prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r+1+\lfloor \frac{s-1}{3} \rfloor} \left((1-\eta)\tilde{v}_{s-4-3\lfloor \frac{s-1}{3} \rfloor}^{(8)} - \zeta\right)}{\theta + \gamma^{4p+r+1} \left((1-\gamma)\hat{v}_{s-3}^{(8)} - \theta\right))} \frac{\theta + \gamma^{4p+r-2} \left((1-\gamma)\hat{v}_{s-3}^{(8)} - \theta\right)}{\zeta + \eta^{4p+r+\lfloor \frac{s-1}{3} \rfloor} \left((1-\eta)\tilde{v}_{s-4-3\lfloor \frac{s-1}{3} \rfloor}^{(8)} - \zeta\right)}, \\ z_{12m+3r+s} &= \alpha^{m+1} z_{3r+s-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r+\lfloor \frac{s-2}{3} \rfloor} \left((1-\gamma)\hat{v}_{s-5-3\lfloor \frac{s-2}{3} \rfloor}^{(8)} - \theta\right)}{\zeta + \eta^{4p+r+1} \left((1-\eta)\tilde{v}_{s-3}^{(8)} - \zeta\right)} \frac{\zeta + \eta^{4p+r-2} \left((1-\eta)\tilde{v}_{s-3}^{(8)} - \zeta\right)}{\theta + \gamma^{4p+r-1+\lfloor \frac{s-2}{3} \rfloor} \left((1-\gamma)\hat{v}_{s-5-3\lfloor \frac{s-2}{3} \rfloor}^{(8)} - \theta\right)}, \end{aligned} \tag{119}$$

if $\gamma \neq 1$, $\eta \neq 1$, and

$$x_{12m+3r+s} = \left(\frac{1}{\alpha^3}\right)^{m+1} x_{3r+s-12} \prod_{p=0}^{m} \frac{\hat{v}_{s-4-3\lfloor \frac{s-1}{3} \rfloor}^{(8)} + \theta\left(4p + r + 1 + \lfloor \frac{s-1}{3} \rfloor\right)}{\hat{v}_{s-4-3\lfloor \frac{s-1}{3} \rfloor}^{(8)} + \theta\left(4p + r + \lfloor \frac{s-1}{3} \rfloor\right)} \frac{\zeta + \eta^{4p+r+\lfloor \frac{s-2}{3} \rfloor} \left((1-\eta)\tilde{v}_{s-5-3\lfloor \frac{s-2}{3} \rfloor}^{(8)} - \zeta\right)}{\zeta + \eta^{4p+r-1+\lfloor \frac{s-2}{3} \rfloor} \left((1-\eta)\tilde{v}_{s-5-3\lfloor \frac{s-2}{3} \rfloor}^{(8)} - \zeta\right)},$$

$$y_{12m+3r+s}=\alpha^{m+1}y_{3r+s-12}\prod_{p=0}^{m}\frac{\zeta+\eta^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\zeta\right)}{\hat{v}^{(8)}_{s-3}+\theta\left(4p+r+1\right)}\frac{\hat{v}^{(8)}_{s-3}+\theta\left(4p+r-2\right)}{\zeta+\eta^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\zeta\right)},\tag{120}$$

$$z_{12m+3r+s}=\alpha^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\theta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\zeta+\eta^{4p+r+1}\left((1-\eta)\tilde{v}^{(8)}_{s-3}-\zeta\right)}\frac{\zeta+\eta^{4p+r-2}\left((1-\eta)\tilde{v}^{(8)}_{s-3}-\zeta\right)}{\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\theta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

if $\gamma=1$, $\eta\neq1$, and

$$x_{12m+3r+s}=\left(\frac{1}{\alpha^3}\right)^{m+1}x_{3r+s-12}\prod_{p=0}^{m}\frac{\theta+\gamma^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\theta\right)}{\theta+\gamma^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\theta\right)}\frac{\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

$$y_{12m+3r+s}=\alpha^{m+1}y_{3r+s-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\theta+\gamma^{4p+r+1}\left((1-\gamma)\hat{v}^{(8)}_{s-3}-\theta\right))}\frac{\theta+\gamma^{4p+r-2}\left((1-\gamma)\hat{v}^{(8)}_{s-3}-\theta\right)}{\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)},\tag{121}$$

$$z_{12m+3r+s}=\alpha^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{\theta+\gamma^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\theta\right)}{\tilde{v}^{(8)}_{s-3}+\zeta\left(4p+r+1\right)}\frac{\tilde{v}^{(8)}_{s-3}+\zeta\left(4p+r-2\right)}{\theta+\gamma^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\theta\right)},$$

if $\gamma\neq1$, $\eta=1$, and

$$x_{12m+3r+s}=\left(\frac{1}{\alpha^3}\right)^{m+1}x_{3r+s-12}\prod_{p=0}^{m}\frac{\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\theta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\theta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}\frac{\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

$$y_{12m+3r+s}=\alpha^{m+1}y_{3r+s-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\hat{v}^{(8)}_{s-3}+\theta\left(4p+r+1\right)}\frac{\hat{v}^{(8)}_{s-3}+\theta\left(4p+r-2\right)}{\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)},\tag{122}$$

$$z_{12m+3r+s}=\alpha^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\theta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\tilde{v}^{(8)}_{s-3}+\zeta\left(4p+r+1\right)}\frac{\tilde{v}^{(8)}_{s-3}+\zeta\left(4p+r-2\right)}{\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\theta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

if $\gamma=1$, $\eta=1$, where $m\in\mathbb{N}_0$, $r\in\{2,3,4,5\}$ and $s\in\{0,1,2\}$.

Case 25: Let $\theta=0$ and $\alpha\beta\gamma\eta\zeta\neq0$. In this case, we get the following system

$$x_n=\frac{y_{n-4}z_{n-5}x_{n-6}}{y_{n-1}z_{n-2}\left(\alpha+\beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}\right)},\quad y_n=\frac{z_{n-4}x_{n-5}y_{n-6}}{\gamma z_{n-1}x_{n-2}},\quad z_n=\frac{x_{n-4}y_{n-5}z_{n-6}}{x_{n-1}y_{n-2}\left(\eta+\zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}\right)},\tag{123}$$

for $n\in\mathbb{N}_0$.

By interchanging y_n, z_n, x_n, γ, η, ζ, α and β instead of x_n, y_n, z_n, α, γ, θ, η and ζ system (111) transforms into system (123). So, the solutions in (119)-(122) turn into the following formulas

$$x_{12m+3r+s}=\gamma^{m+1}x_{3r+s-12}\prod_{p=0}^{m}\frac{\zeta+\eta^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\zeta\right)}{\beta+\alpha^{4p+r+1}\left((1-\alpha)v^{(8)}_{s-3}-\beta\right)}\frac{\beta+\alpha^{4p+r-2}\left((1-\alpha)v^{(8)}_{s-3}-\beta\right)}{\zeta+\eta^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\zeta\right)},$$

$$y_{12m+3r+s}=\left(\frac{1}{\gamma^3}\right)^{m+1}y_{3r+s-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\beta\right)}{\beta+\alpha^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\beta\right)}\frac{\zeta+\eta^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\zeta\right)}{\zeta+\eta^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\zeta\right)},$$

$$z_{12m+3r+s}=\gamma^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\beta\right)}{\zeta+\eta^{4p+r+1}\left((1-\eta)\tilde{v}^{(8)}_{s-3}-\zeta\right)}\frac{\zeta+\eta^{4p+r-2}\left((1-\eta)\tilde{v}^{(8)}_{s-3}-\zeta\right)}{\beta+\alpha^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\beta\right)},$$

if $\alpha\neq 1$, $\eta\neq 1$, and

$$x_{12m+3r+s}=\gamma^{m+1}x_{3r+s-12}\prod_{p=0}^{m}\frac{\zeta+\eta^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\zeta\right)}{v^{(8)}_{s-3}+\beta(4p+r+1)}\frac{v^{(8)}_{s-3}+\beta(4p+r-2)}{\zeta+\eta^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\zeta\right)},$$

$$y_{12m+3r+s}=\left(\frac{1}{\gamma^3}\right)^{m+1}y_{3r+s-12}\prod_{p=0}^{m}\frac{v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\beta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\beta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)}\frac{\zeta+\eta^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\zeta\right)}{\zeta+\eta^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\zeta\right)},$$

$$z_{12m+3r+s}=\gamma^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\zeta+\eta^{4p+r+1}\left((1-\eta)\tilde{v}^{(8)}_{s-3}-\zeta\right)}\frac{\zeta+\eta^{4p+r-2}\left((1-\eta)\tilde{v}^{(8)}_{s-3}-\zeta\right)}{v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)},$$

if $\alpha=1$, $\eta\neq 1$, and

$$x_{12m+3r+s}=\gamma^{m+1}x_{3r+s-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\beta+\alpha^{4p+r+1}\left((1-\alpha)v^{(8)}_{s-3}-\beta\right)}\frac{\beta+\alpha^{4p+r-2}\left((1-\alpha)v^{(8)}_{s-3}-\beta\right)}{\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

$$y_{12m+3r+s}=\left(\frac{1}{\gamma^3}\right)^{m+1}y_{3r+s-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\beta\right)}{\beta+\alpha^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\beta\right)}\frac{\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)},$$

$$z_{12m+3r+s}=\gamma^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\beta\right)}{\tilde{v}^{(8)}_{s-3}+\zeta(4p+r+1)}\frac{\tilde{v}^{(8)}_{s-3}+\zeta(4p+r-2)}{\beta+\alpha^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\beta\right)},$$

if $\alpha\neq 1$, $\eta=1$, and

$$x_{12m+3r+s}=\gamma^{m+1}x_{3r+s-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{v^{(8)}_{s-3}+\beta(4p+r+1)}\frac{v^{(8)}_{s-3}+\beta(4p+r-2)}{\tilde{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

$$y_{12m+3r+s}=\left(\frac{1}{\gamma^3}\right)^{m+1}y_{3r+s-12}\prod_{p=0}^{m}\frac{v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\beta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\beta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)}\frac{\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\tilde{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)},$$

$$z_{12m+3r+s}=\gamma^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\tilde{v}^{(8)}_{s-3}+\zeta(4p+r+1)}\frac{\tilde{v}^{(8)}_{s-3}+\zeta(4p+r-2)}{v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)},$$

if $\alpha=1$, $\eta=1$, where $m\in\mathbb{N}_0$, $r\in\{2,3,4,5\}$ and $s\in\{0,1,2\}$.

Case 26: Let $\zeta=0$ and $\alpha\beta\gamma\theta\eta\neq 0$. In this case, we get the following system

$$x_n=\frac{y_{n-4}z_{n-5}x_{n-6}}{y_{n-1}z_{n-2}\left(\alpha+\beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}\right)},\quad y_n=\frac{z_{n-4}x_{n-5}y_{n-6}}{z_{n-1}x_{n-2}\left(\gamma+\theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}\right)},\quad z_n=\frac{x_{n-4}y_{n-5}z_{n-6}}{\eta x_{n-1}y_{n-2}},\quad (124)$$

for $n \in \mathbb{N}_0$.
By interchanging z_n, x_n, y_n, η, α, β, γ and θ instead of x_n, y_n, z_n, α, γ, θ, η and ζ system (111) transforms into system (124). So, the solutions in (119)-(122) turn into the following formulas

$$x_{12m+3r+s} = \eta^{m+1} x_{3r+s-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \theta\right)}{\beta + \alpha^{4p+r+1}\left((1-\alpha)v^{(8)}_{s-3} - \beta\right)} \frac{\beta + \alpha^{4p+r-2}\left((1-\alpha)v^{(8)}_{s-3} - \beta\right)}{\theta + \gamma^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)},$$

$$y_{12m+3r+s} = \eta^{m+1} y_{3r+s-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \beta\right)}{\theta + \gamma^{4p+r+1}\left((1-\gamma)\hat{v}^{(8)}_{s-3} - \theta\right)} \frac{\theta + \gamma^{4p+r-2}\left((1-\gamma)\hat{v}^{(8)}_{s-3} - \theta\right)}{\beta + \alpha^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \beta\right)},$$

$$z_{12m+3r+s} = \left(\frac{1}{\eta^3}\right)^{m+1} z_{3r+s-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \beta\right)}{\beta + \alpha^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \beta\right)} \frac{\theta + \gamma^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)}{\theta + \gamma^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)},$$

if $\alpha \neq 1$, $\gamma \neq 1$, and

$$x_{12m+3r+s} = \eta^{m+1} x_{3r+s-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \theta\right)}{v^{(8)}_{s-3} + \beta\,(4p+r+1)} \frac{v^{(8)}_{s-3} + \beta\,(4p+r-2)}{\theta + \gamma^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)},$$

$$y_{12m+3r+s} = \eta^{m+1} y_{3r+s-12} \prod_{p=0}^{m} \frac{v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \beta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\theta + \gamma^{4p+r+1}\left((1-\gamma)\hat{v}^{(8)}_{s-3} - \theta\right)} \frac{\theta + \gamma^{4p+r-2}\left((1-\gamma)\hat{v}^{(8)}_{s-3} - \theta\right)}{v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \beta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

$$z_{12m+3r+s} = \left(\frac{1}{\eta^3}\right)^{m+1} z_{3r+s-12} \prod_{p=0}^{m} \frac{v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \beta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \beta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)} \frac{\theta + \gamma^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)}{\theta + \gamma^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)},$$

if $\alpha = 1$, $\gamma \neq 1$, and

$$x_{12m+3r+s} = \eta^{m+1} x_{3r_2+s-12} \prod_{p=0}^{m} \frac{\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \theta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\beta + \alpha^{4p+r+1}\left((1-\alpha)v^{(8)}_{s-3} - \beta\right)} \frac{\beta + \alpha^{4p+r-2}\left((1-\alpha)v^{(8)}_{s-3} - \beta\right)}{\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-2}{3}\rfloor} + \theta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)},$$

$$y_{12m+3r+s} = \eta^{m+1} y_{3r+s-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \beta\right)}{\hat{v}^{(8)}_{s-3} + \theta\,(4p+r+1)} \frac{\hat{v}^{(8)}_{s-3} + \theta\,(4p+r-2)}{\beta + \alpha^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \beta\right)},$$

$$z_{12m+3r+s} = \left(\frac{1}{\eta^3}\right)^{m+1} z_{3r+s-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \beta\right)}{\beta + \alpha^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \beta\right)} \frac{\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \theta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \theta\left(4p+r-1\lfloor\frac{s-2}{3}\rfloor\right)},$$

if $\alpha \neq 1$, $\gamma = 1$, and

$$x_{12m+3r+s} = \eta^{m+1} x_{3r_2+s-12} \prod_{p=0}^{m} \frac{\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \theta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{v^{(8)}_{s-3} + \beta\,(4p+r+1)} \frac{v^{(8)}_{s-3} + \beta\,(4p+r-2)}{\hat{v}^{(8)}_{s-4-3\lfloor\frac{s-2}{3}\rfloor} + \theta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)},$$

$$y_{12m+3r+s} = \eta^{m+1} y_{3r+s-12} \prod_{p=0}^{m} \frac{v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \beta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\hat{v}^{(8)}_{s-3} + \theta\,(4p+r+1)} \frac{\hat{v}^{(8)}_{s-3} + \theta\,(4p+r-2)}{v^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \beta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

$$z_{12m+3r+s}=\left(\frac{1}{\eta^3}\right)^{m+1}z_{3r+s-12}\prod_{p=0}^{m}\frac{v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{v^{(8)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}\frac{\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\theta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\hat{v}^{(8)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\theta\left(4p+r-1\lfloor\frac{s-2}{3}\rfloor\right)},$$

if $\alpha=\gamma=1$, where $m\in\mathbb{N}_0$, $r\in\{2,3,4,5\}$ and $s\in\{0,1,2\}$.

Case 27: Let $\alpha\beta\gamma\theta\eta\zeta\neq 0$. In this case, we get the following system

$$\begin{cases}x_n=&\frac{y_{n-4}z_{n-5}x_{n-6}}{y_{n-1}z_{n-2}\left(\alpha+\beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}\right)},\\ y_n=&\frac{z_{n-4}x_{n-5}y_{n-6}}{z_{n-1}x_{n-2}\left(\gamma+\theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}\right)},\quad n\in\mathbb{N}_0.\\ z_n=&\frac{x_{n-4}y_{n-5}z_{n-6}}{x_{n-1}y_{n-2}\left(\eta+\zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}\right)},\end{cases}\tag{125}$$

Multiplying the first equation in system (125) by $y_{n-1}z_{n-2}x_{n-3}$ for all $n\in\mathbb{N}_0$, the second by $z_{n-1}x_{n-2}y_{n-3}$ for all $n\in\mathbb{N}_0$ and the third by $x_{n-1}y_{n-2}z_{n-3}$ for all $n\in\mathbb{N}_0$, it follows that

$$\begin{cases}x_ny_{n-1}z_{n-2}x_{n-3}=&\frac{x_{n-3}y_{n-4}z_{n-5}x_{n-6}}{\alpha+\beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}},\\ y_nz_{n-1}x_{n-2}y_{n-3}=&\frac{y_{n-3}z_{n-4}x_{n-5}y_{n-6}}{\gamma+\theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}},\quad n\in\mathbb{N}_0.\\ z_nx_{n-1}y_{n-2}z_{n-3}=&\frac{z_{n-3}x_{n-4}y_{n-5}z_{n-6}}{\eta+\zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}},\end{cases}\tag{126}$$

By using the change of variables

$$\begin{cases}x_ny_{n-1}z_{n-2}x_{n-3}=&\frac{1}{v_n^{(9)}},\\ y_nz_{n-1}x_{n-2}y_{n-3}=&\frac{1}{\hat{v}_n^{(9)}},\quad n\geq-3,\\ z_nx_{n-1}y_{n-2}z_{n-3}=&\frac{1}{\tilde{v}_n^{(9)}},\end{cases}\tag{127}$$

system (126) becomes

$$v_n^{(9)}=\alpha v_{n-3}^{(9)}+\beta,\quad \hat{v}_n^{(9)}=\gamma\hat{v}_{n-3}^{(9)}+\theta,\quad \tilde{v}_n^{(9)}=\eta\tilde{v}_{n-3}^{(9)}+\zeta,\quad n\in\mathbb{N}_0.\tag{128}$$

From (4), the solutions of the equations in (128) is given respectively

$$\begin{cases}v_{3m+i}^{(9)}=&\begin{cases}\frac{\beta+\alpha^{m+1}\left((1-\alpha)v_{i-3}^{(9)}-\beta\right)}{1-\alpha}, & \alpha\neq1,\\ v_{i-3}^{(9)}+\beta\,(m+1), & \alpha=1,\end{cases}\\ \hat{v}_{3m+i}^{(9)}=&\begin{cases}\frac{\theta+\gamma^{m+1}\left((1-\gamma)\hat{v}_{i-3}^{(9)}-\theta\right)}{1-\gamma}, & \gamma\neq1,\\ \hat{v}_{i-3}^{(9)}+\theta\,(m+1), & \gamma=1,\end{cases}\\ \tilde{v}_{3m+i}^{(9)}=&\begin{cases}\frac{\zeta+\eta^{m+1}\left((1-\eta)\tilde{v}_{i-3}^{(9)}-\zeta\right)}{1-\eta}, & \eta\neq1,\\ \tilde{v}_{i-3}^{(9)}+\zeta\,(m+1), & \eta=1,\end{cases}\end{cases}\tag{129}$$

for $m\in\mathbb{N}_0$ and $i\in\{0,1,2\}$.
From (127), we gain

$$\begin{cases}x_n=&\frac{\hat{v}_{n-1}^{(9)}\tilde{v}_{n-5}^{(9)}v_{n-9}^{(9)}}{v_n^{(9)}\hat{v}_{n-4}^{(9)}\tilde{v}_{n-8}^{(9)}}x_{n-12},\\ y_n=&\frac{\tilde{v}_{n-1}^{(9)}v_{n-5}^{(9)}\hat{v}_{n-9}^{(9)}}{\hat{v}_n^{(9)}\tilde{v}_{n-4}^{(9)}v_{n-8}^{(9)}}y_{n-12},\quad n\geq6,\\ z_n=&\frac{v_{n-1}^{(9)}\hat{v}_{n-5}^{(9)}\tilde{v}_{n-9}^{(9)}}{\tilde{v}_n^{(9)}v_{n-4}^{(9)}\hat{v}_{n-8}^{(9)}}z_{n-12},\end{cases}\tag{130}$$

and eventually

$$\begin{cases} x_{12m+j} = \dfrac{\hat{v}^{(9)}_{12m+j-1}\tilde{v}^{(9)}_{12m+j-5}v^{(9)}_{12m+j-9}}{v^{(9)}_{12m+j}\hat{v}^{(9)}_{12m+j-4}\tilde{v}^{(9)}_{12m+j-8}} x_{12(m-1)+j}, \\ y_{12m+j} = \dfrac{\tilde{v}^{(9)}_{12m+j-1}v^{(9)}_{12m+j-5}\hat{v}^{(9)}_{12m+j-9}}{\hat{v}^{(9)}_{12m+j}\tilde{v}^{(9)}_{12m+j-4}v^{(9)}_{12m+j-8}} y_{12(m-1)+j}, \\ z_{12m+j} = \dfrac{v^{(9)}_{12m+j-1}\hat{v}^{(9)}_{12m+j-5}\tilde{v}^{(9)}_{12m+j-9}}{\tilde{v}^{(9)}_{12m+j}v^{(9)}_{12m+j-4}\hat{v}^{(9)}_{12m+j-8}} z_{12(m-1)+j}, \end{cases} \tag{131}$$

where $m \in \mathbb{N}_0$ and $j = \overline{6, 17}$.
From (131), we attain

$$\begin{aligned} x_{12m+3r+s} &= x_{3r+s-12} \prod_{p=0}^{m} \frac{\hat{v}^{(9)}_{12p+3r+s-1}\tilde{v}^{(9)}_{12p+3r+s-5}v^{(9)}_{12p+3r+s-9}}{v^{(9)}_{12p+3r+s}\hat{v}^{(9)}_{12p+3r+s-4}\tilde{v}^{(9)}_{12p+3r+s-8}}, \\ y_{12m+3r+s} &= y_{3r+s-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(9)}_{12p+3r+s-1}v^{(9)}_{12p+3r+s-5}\hat{v}^{(9)}_{12p+3r+s-9}}{\hat{v}^{(9)}_{12p+3r+s}\tilde{v}^{(9)}_{12p+3r+s-4}v^{(9)}_{12p+3r+s-8}}, \\ z_{12m+3r+s} &= z_{3r+s-12} \prod_{p=0}^{m} \frac{v^{(9)}_{12p+3r+s-1}\hat{v}^{(9)}_{12p+3r+s-5}\tilde{v}^{(9)}_{12p+3r+s-9}}{\tilde{v}^{(9)}_{12p+3r+s}v^{(9)}_{12p+3r+s-4}\hat{v}^{(9)}_{12p+3r+s-8}}, \end{aligned} \tag{132}$$

where $m \in \mathbb{N}_0$, $r \in \{2, 3, 4, 5\}$ and $s \in \{0, 1, 2\}$.
From (129) and (132), we obtain

$$\begin{aligned} x_{12m+3r+s} = x_{3r+s-12} \prod_{p=0}^{m} & \frac{\theta + \gamma^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \theta\right)}{\beta + \alpha^{4p+r+1}\left((1-\alpha)v^{(9)}_{s-3} - \beta\right)} \frac{\zeta + \eta^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \zeta\right)}{\theta + \gamma^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \theta\right)} \\ & \times \frac{\beta + \alpha^{4p+r-2}\left((1-\alpha)v^{(9)}_{s-3} - \beta\right)}{\zeta + \eta^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \zeta\right)}, \\ y_{12m+3r+s} = y_{3r+s-12} \prod_{p=0}^{m} & \frac{\zeta + \eta^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \zeta\right)}{\theta + \gamma^{4p+r+1}\left((1-\gamma)\hat{v}^{(9)}_{s-3} - \theta\right)} \frac{\beta + \alpha^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \beta\right)}{\zeta + \eta^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \zeta\right)} \\ & \times \frac{\theta + \gamma^{4p+r-2}\left((1-\gamma)\hat{v}^{(9)}_{s-3} - \theta\right)}{\beta + \alpha^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \beta\right)}, \\ z_{12m+3r+s} = z_{3r+s-12} \prod_{p=0}^{m} & \frac{\beta + \alpha^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \beta\right)}{\zeta + \eta^{4p+r+1}\left((1-\eta)\tilde{v}^{(9)}_{s-3} - \zeta\right)} \frac{\theta + \gamma^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)}{\beta + \alpha^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \beta\right)} \\ & \times \frac{\zeta + \eta^{4p+r-2}\left((1-\eta)\tilde{v}^{(9)}_{s-3} - \zeta\right)}{\theta + \gamma^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)}, \end{aligned}$$

if $\alpha \neq 1$, $\gamma \neq 1$, $\eta \neq 1$, and

$$x_{12m+3r+s} = x_{3r+s-12} \prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \theta\right)}{v^{(9)}_{s-3} + \beta\,(4p+r+1)} \frac{\zeta + \eta^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \zeta\right)}{\theta + \gamma^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \theta\right)}$$

$$\times \frac{v^{(9)}_{s-3} + \beta\,(4p+r-2)}{\zeta + \eta^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \zeta\right)},$$

$$y_{12m+3r+s} = y_{3r+s-12}\prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \zeta\right)}{\theta + \gamma^{4p+r+1}\left((1-\gamma)\hat{v}^{(9)}_{s-3} - \theta\right)} \frac{v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \beta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\zeta + \eta^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \zeta\right)}$$

$$\times \frac{\theta + \gamma^{4p+r-2}\left((1-\gamma)\hat{v}^{(9)}_{s-3} - \theta\right)}{v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \beta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

$$z_{12m+3r+s} = z_{3r+s-12}\prod_{p=0}^{m} \frac{v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \beta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\zeta + \eta^{4p+r+1}\left((1-\eta)\tilde{v}^{(9)}_{s-3} - \zeta\right)} \frac{\theta + \gamma^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)}{v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \beta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$

$$\times \frac{\zeta + \eta^{4p+r-2}\left((1-\eta)\tilde{v}^{(9)}_{s-3} - \zeta\right)}{\theta + \gamma^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)},$$

if $\alpha = 1, \gamma \neq 1, \eta \neq 1$, and

$$x_{12m+3r+s} = x_{3r+s-12}\prod_{p=0}^{m} \frac{\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \theta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\beta + \alpha^{4p+r+1}\left((1-\alpha)v^{(9)}_{s-3} - \beta\right)} \frac{\zeta + \eta^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \zeta\right)}{\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \theta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$

$$\times \frac{\beta + \alpha^{4p+r-2}\left((1-\alpha)v^{(9)}_{s-3} - \beta\right)}{\zeta + \eta^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \zeta\right)},$$

$$y_{12m+3r+s} = y_{3r+s-12}\prod_{p=0}^{m} \frac{\zeta + \eta^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \zeta\right)}{\hat{v}^{(9)}_{s-3} + \theta\,(4p+r+1)} \frac{\beta + \alpha^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \beta\right)}{\zeta + \eta^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \zeta\right)}$$

$$\times \frac{\hat{v}^{(9)}_{s-3} + \theta\,(4p+r-2)}{\beta + \alpha^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \beta\right)},$$

$$z_{12m+3r+s} = z_{3r+s-12}\prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \beta\right)}{\zeta + \eta^{4p+r+1}\left((1-\eta)\tilde{v}^{(9)}_{s-3} - \zeta\right)} \frac{\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \theta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\beta + \alpha^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \beta\right)}$$

$$\times \frac{\zeta + \eta^{4p+r-2}\left((1-\eta)\tilde{v}^{(9)}_{s-3} - \zeta\right)}{\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \theta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

if $\alpha \neq 1, \gamma = 1, \eta \neq 1$, and

$$x_{12m+3r+s} = x_{3r+s-12}\prod_{p=0}^{m} \frac{\theta + \gamma^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \theta\right)}{\beta + \alpha^{4p+r+1}\left((1-\alpha)v^{(9)}_{s-3} - \beta\right)} \frac{\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \zeta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\theta + \gamma^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \theta\right)}$$

$$\times \frac{\beta+\alpha^{4p+r-2}\left((1-\alpha)v^{(9)}_{s-3}-\beta\right)}{\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

$$y_{12m+3r+s}=y_{3r+s-12}\prod_{p=0}^{m}\frac{\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\theta+\gamma^{4p+r+1}\left((1-\gamma)\hat{v}^{(9)}_{s-3}-\theta\right)}\frac{\beta+\alpha^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\beta\right)}{\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$

$$\times\frac{\theta+\gamma^{4p+r-2}\left((1-\gamma)\hat{v}^{(9)}_{s-3}-\theta\right)}{\beta+\alpha^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\beta\right)},$$

$$z_{12m+3r+s}=z_{3r+s-12}\prod_{p=0}^{m}\frac{\beta+\alpha^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\beta\right)}{\tilde{v}^{(9)}_{s-3}+\zeta\left(4p+r+1\right)}\frac{\theta+\gamma^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\theta\right)}{\beta+\alpha^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\beta\right)}$$

$$\times\frac{\tilde{v}^{(9)}_{s-3}+\zeta\left(4p+r-2\right)}{\theta+\gamma^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\theta\right)},$$

if $\alpha\neq1$, $\gamma\neq1$, $\eta=1$, and

$$x_{12m+3r+s}=x_{3r+s-12}\prod_{p=0}^{m}\frac{\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\theta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{v^{(9)}_{s-3}+\beta\left(4p+r+1\right)}\frac{\zeta+\eta^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\zeta\right)}{\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\theta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$

$$\times\frac{v^{(9)}_{s-3}+\beta\left(4p+r-2\right)}{\zeta+\eta^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}-\zeta\right)},$$

$$y_{12m+3r+s}=y_{3r+s-12}\prod_{p=0}^{m}\frac{\zeta+\eta^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\zeta\right)}{\hat{v}^{(9)}_{s-3}+\theta\left(4p+r+1\right)}\frac{v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\beta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\zeta+\eta^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\eta)\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\zeta\right)}$$

$$\times\frac{\hat{v}^{(9)}_{s-3}+\theta\left(4p+r-2\right)}{v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\beta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

$$z_{12m+3r+s}=z_{3r+s-12}\prod_{p=0}^{m}\frac{v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\zeta+\eta^{4p+r+1}\left((1-\eta)\tilde{v}^{(9)}_{s-3}-\zeta\right)}\frac{\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\theta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$

$$\times\frac{\zeta+\eta^{4p+r-2}\left((1-\eta)\tilde{v}^{(9)}_{s-3}-\zeta\right)}{\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\theta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

if $\alpha=1$, $\gamma=1$, $\eta\neq1$, and

$$x_{12m+3r+s}=x_{3r+s-12}\prod_{p=0}^{m}\frac{\theta+\gamma^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\theta\right)}{v^{(9)}_{s-3}+\beta\left(4p+r+1\right)}\frac{\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\theta+\gamma^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}-\theta\right)}$$

$$\times\frac{v^{(9)}_{s-3}+\beta\left(4p+r-2\right)}{\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\zeta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

$$y_{12m+3r+s} = y_{3r+s-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \zeta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\theta + \gamma^{4p+r+1}\left((1-\gamma)\hat{v}^{(9)}_{s-3} - \theta\right)} \frac{v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \beta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \zeta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$
$$\times \frac{\theta + \gamma^{4p+r-2}\left((1-\gamma)\hat{v}^{(9)}_{s-3} - \theta\right)}{v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \beta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)}, \tag{133}$$
$$z_{12m+3r+s} = z_{3r+s-12} \prod_{p=0}^{m} \frac{v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \beta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\tilde{v}^{(9)}_{s-3} + \zeta(4p+r+1)} \frac{\theta + \gamma^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)}{v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \beta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$
$$\times \frac{\tilde{v}^{(9)}_{s-3} + \zeta(4p+r-2)}{\theta + \gamma^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\gamma)\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \theta\right)},$$

if $\alpha = 1, \gamma \neq 1, \eta = 1$, and

$$x_{12m+3r+s} = x_{3r+s-12} \prod_{p=0}^{m} \frac{\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \theta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\beta + \alpha^{4p+r+1}\left((1-\alpha)v^{(9)}_{s-3} - \beta\right)} \frac{\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \zeta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \theta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$
$$\times \frac{\beta + \alpha^{4p+r-2}\left((1-\alpha)v^{(9)}_{s-3} - \beta\right)}{\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \zeta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$
$$y_{12m+3r+s} = y_{3r+s-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \zeta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\hat{v}^{(9)}_{s-3} + \theta(4p+r+1)} \frac{\beta + \alpha^{4p+r+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \beta\right)}{\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \zeta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$
$$\times \frac{\hat{v}^{(9)}_{s-3} + \theta(4p+r-2)}{\beta + \alpha^{4p+r-1+\lfloor\frac{s-2}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} - \beta\right)},$$
$$z_{12m+3r+s} = z_{3r+s-12} \prod_{p=0}^{m} \frac{\beta + \alpha^{4p+r+1+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \beta\right)}{\tilde{v}^{(9)}_{s-3} + \zeta(4p+r+1)} \frac{\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \theta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\beta + \alpha^{4p+r+\lfloor\frac{s-1}{3}\rfloor}\left((1-\alpha)v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} - \beta\right)}$$
$$\times \frac{\tilde{v}^{(9)}_{s-3} + \zeta(4p+r-2)}{\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \theta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

if $\alpha \neq 1, \gamma = 1, \eta = 1$, and

$$x_{12m+3r+s} = x_{3r+s-12} \prod_{p=0}^{m} \frac{\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \theta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{v^{(9)}_{s-3} + \beta(4p+r+1)} \frac{\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \zeta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\hat{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \theta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$
$$\times \frac{v^{(9)}_{s-3} + \beta(4p+r-2)}{\tilde{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \zeta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$
$$y_{12m+3r+s} = y_{3r+s-12} \prod_{p=0}^{m} \frac{\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \zeta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\hat{v}^{(9)}_{s-3} + \theta(4p+r+1)} \frac{v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor} + \beta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{\tilde{v}^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor} + \zeta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$

$$\times\frac{\hat{v}^{(9)}_{s-3}+\theta\left(4p+r-2\right)}{v^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\beta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

$$z_{12m+3r+s}=z_{3r+s-12}\prod_{p=0}^{m}\frac{v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+1+\lfloor\frac{s-1}{3}\rfloor\right)}{\tilde{v}^{(9)}_{s-3}+\zeta\left(4p+r+1\right)}\frac{\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\theta\left(4p+r+\lfloor\frac{s-2}{3}\rfloor\right)}{v^{(9)}_{s-4-3\lfloor\frac{s-1}{3}\rfloor}+\beta\left(4p+r+\lfloor\frac{s-1}{3}\rfloor\right)}$$
$$\times\frac{\tilde{v}^{(9)}_{s-3}+\zeta\left(4p+r-2\right)}{\hat{v}^{(9)}_{s-5-3\lfloor\frac{s-2}{3}\rfloor}+\theta\left(4p+r-1+\lfloor\frac{s-2}{3}\rfloor\right)},$$

if $\alpha=\gamma=\eta=1$, where $m\in\mathbb{N}_0$, $r\in\{2,3,4,5\}$ and $s\in\{0,1,2\}$.

Theorem 2.1. *Assume that $\alpha\neq0$, $\beta\neq0$, $\gamma\neq0$, $\theta\neq0$, $\eta\neq0$, $\zeta\neq0$. The forbidden set of the initial values for system* (12) *is given by sets*

$$\mathbb{F}=\bigcup_{m\in\mathbb{N}_0}\bigcup_{i=3}^{8}\left\{\frac{1}{x_{i-3}y_{i-4}z_{i-5}x_{i-6}}=f^{-m-1}\left(-\frac{\beta}{\alpha}\right),\ or\ \frac{1}{y_{i-3}z_{i-4}x_{i-5}y_{i-6}}=g^{-m-1}\left(-\frac{\theta}{\gamma}\right),\ or\ \frac{1}{z_{i-3}x_{i-4}y_{i-5}z_{i-6}}=h^{-m-1}\left(-\frac{\zeta}{\eta}\right)\right\}\tag{134}$$

and

$$\left\{\vec{\mathbb{F}}:x_{-p}=0\quad or\quad y_{-p}=0\quad or\quad z_{-p}=0,\quad p\in\{1,2,3,4,5,6\}\right\}$$

where $\vec{\mathbb{F}}=(x_{-6},x_{-5},x_{-4},x_{-3},x_{-2},x_{-1},y_{-6},y_{-5},y_{-4},y_{-3},y_{-2},y_{-1},z_{-6},z_{-5},z_{-4},z_{-3},z_{-2},z_{-1})$.

Proof. We have gained the set

$$\left\{\vec{\mathbb{F}}:x_{-p}=0\quad\text{or}\quad y_{-p}=0\quad\text{or}\quad z_{-p}=0,\quad p\in\{1,2,3,4,5,6\}\right\}$$

belongs to the forbidden set of the initial values for system (12) at the beginning of the section 2. If $x_{-j}\neq0$, $y_{-j}\neq0$ and $z_{-j}\neq0$, $j\in\{1,2,3,4,5,6\}$, system (12) is undefined if and only if

$$\begin{cases}\alpha+\beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}=0,\\ \gamma+\theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}=0,\quad n\in\mathbb{N}_0.\\ \eta+\zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}=0,\end{cases}$$

By using the change of variables (127), the above conditions can be written

$$v_{n-3}=-\frac{\beta}{\alpha},\quad \hat{v}_{n-3}=-\frac{\theta}{\gamma},\quad \tilde{v}_{n-3}=-\frac{\zeta}{\eta},\quad n\in\mathbb{N}_0.\tag{135}$$

Thus, the solutions of system (128) can be expressed as

$$v_{3m+1}=f^{m+1}\left(v_{i-3}\right),\quad \hat{v}_{3m+1}=g^{m+1}\left(\hat{v}_{i-3}\right),\quad \tilde{v}_{3m+1}=h^{m+1}\left(\tilde{v}_{i-3}\right),\tag{136}$$

where $m\in\mathbb{N}_0$, $i\in\{0,1,2\}$, $f(r)=\alpha r+\beta$, $g(r)=\gamma r+\theta$, $h(r)=\eta r+\zeta$. By using (135) and (136), we get

$$v_{i-3}=f^{-m-1}\left(-\frac{\beta}{\alpha}\right),\quad \hat{v}_{i-3}=g^{-m-1}\left(-\frac{\theta}{\gamma}\right),\quad \tilde{v}_{i-3}=h^{-m-1}\left(-\frac{\zeta}{\eta}\right),\tag{137}$$

where $m\in\mathbb{N}_0$, $i\in\{0,1,2\}$, $f^{-1}(r)=\frac{r-\beta}{\alpha}$, $g^{-1}(r)=\frac{r-\theta}{\gamma}$, $h^{-1}(r)=\frac{r-\zeta}{\eta}$.
This means that if one of the conditions in (137) holds, then $6m-th$ iteration or $6(m+1)-th$ iteration can not be calculated. Consequently, (134) is attained. □

3. Conclusion

In this study, we gain the solutions of the following system of difference equations,

$$\begin{cases} x_n = \dfrac{y_{n-4}z_{n-5}x_{n-6}}{y_{n-1}z_{n-2}\left(\alpha+\beta x_{n-3}y_{n-4}z_{n-5}x_{n-6}\right)}, \\ y_n = \dfrac{z_{n-4}x_{n-5}y_{n-6}}{z_{n-1}x_{n-2}\left(\gamma+\theta y_{n-3}z_{n-4}x_{n-5}y_{n-6}\right)}, \\ z_n = \dfrac{x_{n-4}y_{n-5}z_{n-6}}{x_{n-1}y_{n-2}\left(\eta+\zeta z_{n-3}x_{n-4}y_{n-5}z_{n-6}\right)}, \end{cases} \quad n \in \mathbb{N}_0,$$

where the initial values x_{-p}, y_{-p}, z_{-p} for $p = \overline{1,6}$ and the parameters α, β, γ, θ, η, ζ are real numbers. Firstly, we solved the mentioned system depending on whether the parameters are equal to zero or non-zero. In addition, the solutions of the aforementioned system are obtained in closed form. Finally, we also describe the forbidden set of the solutions of this system of difference equations.

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