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On 2-absorbing ideals of MV-algebras

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Abstract. The concept of 2-absorbing ideals in MV-algebras, which generalizes the notion of prime ideals, is introduced in this paper. We present several results related to 2-absorbing ideals, provide examples, and explore some of properties. One notable result is that the intersection of two prime ideals, which is not necessarily a prime ideal, is shown to be a 2-absorbing ideal. The number of minimal prime ideals contained in a 2-absorbing ideal under different conditions are computed. Furthermore, we demonstrate that if every 2-absorbing ideal is maximal, then the MV-algebra can have at most one prime ideal. Finally, these new ideals are used to classify the ideals of MV-algebra.

1. Introduction

MV-algebras were introduced and studied by Chang in 1958 as an algebraic counterpart of the Lukasiewicz infinite valued propositional logic [3, 4]. For further results on MV-algebras see [3, 5, 7, 13], and for a deeper understanding of ideals, we refer to [6, 10, 11]. Chang also introduced the concept of prime ideals in MV-algebras, which play a crucial role since every proper ideal can be expressed as an intersection of prime ideals, including itself [13]. Studying ideals is important for a better understanding of MV-algebras, and for this purpose, various ideals have been introduced in this structure. This motivated us to introduce a new ideal, study it, and obtain a classification for these ideals. Various generalizations of prime ideals have since been studied, and in this paper, we introduce the concept of 2-absorbing ideals. The aim of this article is to define 2-absorbing ideals and demonstrate their distinctiveness through examples. It is known that the intersection of two prime ideals is not necessarily prime. In this paper, it is proved that their intersection is 2-absorbing. Moreover, it is shown that this result does not hold for the intersection of three prime ideals. We examine the connection of these ideals with other ideals, and we also represent this connection in the form of a diagram. By utilizing these ideals, the number of minimal prime ideals contained within a 2-absorbing ideal is investigated. It is established that for every 2-absorbing ideal, the maximum number of associated minimal prime ideals is two. Additionally, if a 2-absorbing ideal is not a prime ideal, it will also be found to have exactly two minimal prime ideals. Furthermore, we demonstrate that if the collection of 2-absorbing ideals coincides with the collection of maximal ideals in an MV-algebra,

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then that algebra can have at most one prime ideal. When considering the case where A/I is locally finite, it is confirmed that I is a 2-absorbing ideal, while an example is illustrated to show that the converse does not generally hold. Additionally, we show that every proper ideal can be expressed as an intersection of 2-absorbing ideals that contain it. 2-absorbing ideals in MV-chains are also examined, and it is proved that every proper ideal in this context is indeed a 2-absorbing ideal. Finally, the conditions under which I/S is a 2-absorbing ideal in A/S are studied.

2. Preliminaries

We recollect some definitions and results which will be used in the sequel:

Definition 2.1. ([4]) *An MV-algebra is a structure* $(A, \oplus, *, 0)$ *of type* $(2, 1, 0)$ *such that the following axioms hold, for each* $a, b \in A$:
(MV1) $(A, \oplus, 0)$ *is an abelian monoid;*
(MV2) $(a^*)^* = a$;
(MV3) $0^* \oplus a = 0^*$;
(MV4) $(a^* \oplus b)^* \oplus b = (b^* \oplus a)^* \oplus a$.

Define $1 = 0^*$ and the auxiliary operation $\odot$ which are as follows:

$$a \odot b = (a^* \oplus b^*)^*.$$

Two operations $\vee$ and $\wedge$ are defined on A :

$$a \vee b = a \oplus (a^* \odot b) = b \oplus (a \odot b^*) \quad \text{and} \quad a \wedge b = a \odot (a^* \oplus b) = b \odot (b^* \oplus a).$$

Also, for any two elements $a, b \in A$, $a \leq b$ iff $a^* \oplus b = 1$ iff $a \odot b^* = 0$. Obviously, $\leq$ is a partial order on A which is called the natural order on A. We say that an MV-algebra A is an MV-chain if it is linearly ordered relative to natural order. Boolean algebras are just the MV-algebras obeying the additional equation $a \oplus a = a$, for all $a \in A$.
The element $a \in M$ is said to have order n and is written as $ord(a) = n$ if n is the smallest natural number for which $na = 1$. We say that the element a has a finite order and write $ord(a) < \infty$. An MV-algebra A is locally finite if every nonzero element of A has finite order.
Throughout this paper, A is an MV-algebra.

Remark 2.2. ([13]) *If A is locally finite, then A is a chain.*

Theorem 2.3. ([13]) *If* $a, a_1, a_2, ..., a_n$ *are elements of A, then the following hold:*
(i) $a \wedge (a_1 \oplus a_2 \oplus ... \oplus a_n) \leq (a \wedge a_1) \oplus (a \wedge a_2) \oplus ... \oplus (a \wedge a_n)$.
(ii) $a \wedge (a_1 \vee a_2 \vee ... \vee a_n) = (a \wedge a_1) \vee ... \vee (a \wedge a_n)$.

Definition 2.4. ([5, 7, 11, 13]) *An ideal of A is a nonempty subset I of A which is closed under* $\oplus$ *and such that if* $b \in I$, $a \in A$ *and* $a \leq b$, *then* $a \in I$.
We denote the set of all ideals of A by $Id(A)$.
A proper ideal I of A is called:
• *A prime ideal, if* $a \wedge b \in I$ *implies that* $a \in I$ *or* $b \in I$, *for each* $a, b \in A$.
We denote the set of all prime ideals of A by $Spec(A)$.
• *A primary ideal, if* $a \odot b \in I$, *then there exists* $n \in \mathbb{N}$ *such that* $a^n \in I$ *or* $b^n \in I$, *for each* $a, b \in A$.
• *An obstinate ideal, if* $a, b \notin I$ *imply* $a \odot b^* \in I$ *and* $b \odot a^* \in I$, *for all* $a, b \in A$.
• *A quasi implicative ideal, if for any* $a \in A$ *such that* $a^n \in I$ *for some* $n \geq 1$, *then* $a \in I$.
• *An implicative ideal, if for any* $a, b, c \in A$ *such that* $c \odot (b^* \odot a^*) \in I$ *and* $b \odot a^* \in I$, *then* $c \odot a^* \in I$.
• *A Boolean ideal, if* $a \wedge a^* \in I$, *for all* $a \in A$.
• *A maximal ideal if and only if whenever J is an ideal such that* $I \subseteq J \subseteq A$, *then either* $J = I$ *or* $J = A$.
• *A prime ideal P of A is called a minimal prime ideal of I whenever:*
(i) $I \subseteq P$;
(ii) If there exists $Q \in Spec(A)$ *such that* $I \subseteq Q \subseteq P$, *then* $P = Q$.
We denote the set of all minimal prime ideals of I by $Min(I)$.

Theorem 2.5. ([7]) (i) *Let I be an obstinate ideal of A. Then I is a maximal ideal of A.*
(ii) *If I is an implicative and maximal ideal, then I is an obstinate ideal.*

Remark 2.6. ([13]) *If I is an ideal of A, then* $a \oplus b \in I$ *iff* $a \vee b \in I$.

Lemma 2.7. ([13]) (i) *Let* $X \subseteq A$. *Denote by* $(X]$ *the ideal generated by X. Then we have*
$(X] = \{a \in A \mid a \leqslant x_1 \oplus x_2 \oplus ... \oplus x_n, \text{ for some } n \in \mathbb{N} \text{ and } x_1, ..., x_n \in X\}$.
In particular, $(a] = \{x \in A \mid x \leqslant na, \text{ for some } n \in \mathbb{N}\}$.
(ii) *For* $I, J \in Id(A)$, *we put*

$$I \wedge J = I \cap J \quad I \vee J = (I \cup J] = \{x \in A \mid x \leq a \oplus b, \text{ for some } a \in I \text{ and } b \in J\}.$$

(iii) *If* $a, b \in A$, *then* $(a] \cap (b] = (a \wedge b]$ *and* $(a] \vee (b] = (a \oplus b]$.
For $I \in Id(A)$ *and* $a \in A \setminus I$ *we denote by* $I(a) = I \vee (a] = (I \cup \{a\}]$.
For $I(a)$ *we have the next characterization:*
$I(a) = \{x \in A \mid x \leq y \oplus na, \text{for some } y \in I \text{ and integer } n \geq 0\}$.

Theorem 2.8. ([13]) *For any A, the following are equivalent:*
(i) *A is an MV-chain;*
(ii) *Any proper ideal of A is prime;*
(iii) $\{0\}$ *is a prime ideal;*
(iv) $Spec(A)$ *is linearly ordered.*

Theorem 2.9. ([13]) *Every proper ideal of A is contained in a maximal ideal of A.*

Theorem 2.10. ([13]) *Let* $I \in Id(A)$. *Then* $I = \cap\{P \in Spec(A) \mid I \subseteq P\}$.

Definition 2.11. ([13]) *Let A and B be MV-algebras. A function* $f : A \longrightarrow B$ *is a morphism of MV-algebras if and only if it satisfies the following conditions, for every* $a, b \in A$:
(i) $f(0) = 0$;
(ii) $f(a \oplus b) = f(a) \oplus f(b)$;
(iii) $f(a^*) = (f(a))^*$.

Definition 2.12. ([9]) *Let X be a nonempty subset of A.* $Ann_A(X)$ *is the annihilator of X defined by:* $Ann_A(X) = \{a \in A \mid a \wedge x = 0, \forall x \in X\}$

Definition 2.13. ([2]) *Let X be a nonempty subset of A. For an ideal I of A, the set*

$$(I : X) = \{a \in A \mid a \wedge x \in I, \text{for all } x \in X\}.$$

• $(I : X)$ *is clearly an ideal of A and if* $I = 0$, *then* $(0 : X) = Ann(X)$.

Theorem 2.14. ([2]) *Let I be a proper ideal of A and* $P \in Spec(A)$ *such that* $I \subseteq P$. *Then there exists* $P^* \in Min(I)$ *such that* $P^* \subseteq P$.

Definition 2.15. ([13]) *A nonempty subset S of A is called* $\wedge$*-closed system in A if* $1 \in S$ *and* $a, b \in S$ *imply that* $a \wedge b \in S$.

We denote by $S(A)$ the set of all $\wedge$-closed systems of A (clearly 1, $A \in S(A)$). For $S \in S(A)$ in A, we consider the relation θ_S defined by:

$$(a, b) \in \theta_S \text{ if and only if there exists } e \in S \cap B(A) \text{ such that } a \wedge e = b \wedge e.$$

Lemma 2.16. ([13]) θ_S *is a congruence on A.*

For $a \in A$, we denote by a/S the equivalence class of a relative to θ_S and $A/S = A/\theta_S$.
By $P_S : A \to A/S$, we denote the canonical map defined by $P_S(a) = a/S$, for every $a \in A$. Clearly, $0 = 0/S, 1 = 1/S$ *in* A/S and for every $a, b \in A$,

$$a/S \oplus b/S = (a \oplus b)/S \text{ and } (a/S)^* = a^*/S.$$

So, P_S is an onto morphism of MV-algebras ([13]).

Theorem 2.17. ([13]) *For a proper ideal $P \in Id(A)$, A/P is a chain if and only if $P \in Spec(A)$.*

Definition 2.18. ([1]) *Let $C(X)$ be the set of all continuous functions on topological space X to the real interval $[0, 1]$. For every $f, g \in C(X)$, define*

$$(f \oplus g)(x) = f(x) \oplus g(x) = \min\{1, f(x) + g(x)\}, \text{ for all } x \in X;$$
$$f^*(x) = (f(x))^* = 1 - f(x), \text{ for all } x \in X;$$
$$0(x) = 0, \text{ for all } x \in X.$$

The structure $(C(X), \oplus, *, 0)$ is called the MV-algebra of continuous functions.

Let $f, g \in C(X)$. Define

$$Z(f) = \{x \in X : f(x) = 0\}$$

It is clear $Z(f \wedge g) = Z(f) \cup Z(g)$ and $(f \wedge g)(x) = min\{f(x), g(x)\}$, for every $x \in X$.

Theorem 2.19. ([1]) *Let X be a topological space with $x \in X$. Then $M_x = \{f \in C(X) : f(x) = 0\}$ is a maximal ideal of $C(X)$.*

3. A generalization of prime ideals in MV-algebras

In this section, we introduce the concept of a 2-absorbing ideal and explore its relationship with prime ideals. Several properties of these ideals are stated and proved, and an equivalent definition is provided.

Definition 3.1. *A proper ideal I of A is called a 2-absorbing ideal if $a, b, c \in A$ such that $a \wedge b \wedge c \in I$, then $a \wedge b \in I$ or $a \wedge c \in I$ or $b \wedge c \in I$.*

Example 3.2. (i) *Let $A = \{0, a, b, c, d, 1\}$. Where $0 < a, b < c < 1$, $0 < b < d < 1$, with the diagram below (see Figure 1):*

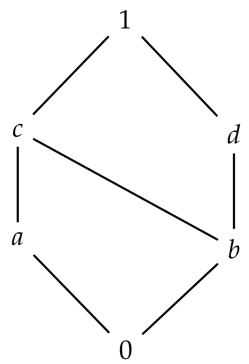

Figure 1: A nonlinearly ordered

Define $\oplus$, $\odot$ and $*$ as follows:

$\oplus$	0	a	b	c	d	1
0	0	a	b	c	d	1
a	a	a	c	c	1	1
b	b	c	d	1	d	1
c	c	c	1	1	1	1
d	d	1	d	1	d	1
1	1	1	1	1	1	1

$\odot$	0	a	b	c	d	1
0	0	0	0	0	0	0
a	0	a	0	a	0	a
b	0	0	0	0	b	b
c	0	a	0	a	b	c
d	0	0	b	b	d	d
1	0	a	b	c	d	1

$*$	0	a	b	c	d	1
	1	d	c	b	a	0

Then $(A, \oplus, *, 0)$ *is an MV-algebra* ([12]). $I_0 = \{0\}$, $I_1 = \{0, a\}$, $I_2 = \{0, b, d\}$, *and* $I_4 = A$ *are ideals.* I_0, I_1 *and* I_2 *are* 2*-absorbing ideals. Since* I_4 *is not proper, so it is not a* 2*-absorbing ideal.*
(ii) *Let* $X = \mathbb{R}$. *Obviously,* $I = \{k \in C(X) : (1, 5) \subseteq Z(k)\}$ *is an ideal of* $C(X)$. *Put*

$$f(x) = \begin{cases} -x+1 & x \in (0,1) \\ x-3 & x \in (3,4) \\ 1 & x \in (-\infty, 0] \cup [4, \infty) \\ 0 & x \in [1,3] \end{cases}$$

$$g(x) = \begin{cases} x-5 & x \in (5,6) \\ -x+4 & x \in (3,4) \\ 1 & x \in (-\infty, 3] \cup [6, \infty) \\ 0 & x \in [4,5] \end{cases}$$

$$h(x) = \begin{cases} x-4 & x \in (4,5) \\ -x+3 & x \in (2,3) \\ 1 & x \in (-\infty, 2] \cup [5, \infty) \\ 0 & x \in [3,4] \end{cases}$$

Now, we have $Z(f \wedge g \wedge h) = [1, 5]$. *Thus* $f \wedge g \wedge h \in I$.
On the other hand, $Z(f \wedge g) = [1, 3] \cup [4, 5], Z(f \wedge h) = [1, 4]$ *and* $Z(g \wedge h) = [3, 5]$.
It is clear that $f \wedge g \notin I, f \wedge h \notin I$ *and* $g \wedge h \notin I$. *Hence* I *is not a* 2*-absorbing ideal of* $C(X)$.
(iii) *Let* $X = \mathbb{R}$. *Put*

$$f(x) = \begin{cases} 0 & x \in (-\infty, 0] \\ x & x \in (0,1) \\ 1 & x \in [1, \infty) \end{cases}$$

$$g(x) = \begin{cases} 1 & x \in (-\infty, 0] \\ 0 & x \in [1, \infty) \\ -x+1 & x \in (0,1) \end{cases}$$

$$h(x) = \begin{cases} 1 & x \in (-\infty, -1] \cup [2, \infty) \\ 0 & x \in [0,1] \\ -x & x \in (-1,0) \\ x-1 & x \in (1,2) \end{cases}$$

Obviously, $f \wedge g \wedge h = 0$ *but* $f \wedge g, g \wedge h$ *and* $f \wedge h$ *do not belong to the zero ideal of* $C(X)$. *The zero ideal is not a* 2*-absorbing ideal.*

Proposition 3.3. (i) *If* P *is a prime ideal of* A, *then* P *is a* 2*-absorbing ideal.*
(ii) *Let* I *be a primary ideal and a quasi implicative ideal of* A. *Then* I *is a* 2*-absorbing ideal.*
(iii) *Let* I *be a proper ideal of* A. *Then* $I = \bigcap\{P \mid P$ *is a* 2*-absorbing ideal and* $I \subseteq P\}$.
(iv) *If* A/I *is locally finite, then* I *is a* 2*-absorbing ideal.*
(v) *If* $A \setminus \{1\}$ *is an ideal of* A, *then* $A \setminus \{1\}$ *is a* 2*-absorbing ideal.*

Proof. (i) Suppose $a, b, c \in A$ with $a \wedge b \wedge c \in P$ and $a \wedge c, b \wedge c \notin P$. Then $(a \wedge b) \wedge c \in P$. Since P is a prime ideal, so $a \wedge b \in P$ or $c \in P$. If $a \wedge b \in P$, then we are done. If $c \in P$, since $a \wedge c, b \wedge c \leq c$ and P is an ideal, so $a \wedge c, b \wedge c \in P$, which is a contradiction.
(ii) Suppose $a \wedge b \wedge c \in I$, for some $a, b, c \in A$. Obviously, $(a \wedge b) \odot c \in I$ and since I is primary, so there exists

$n \in \mathbb{N}$ such that $(a \wedge b)^n \in I$ or $c^n \in I$. If $(a \wedge b)^n \notin I$, for all $n \in \mathbb{N}$, then $a \wedge b \notin I$. Now, we show that $a \wedge c \in I$ or $b \wedge c \in I$. As $c^n \in I$ and I is a quasi implicative ideal, hence $c \in I$. By ideal property, we deduce that $a \wedge c \in I$ and $b \wedge c \in I$. Thus I is a 2-absorbing ideal.
(*iii*) Always, $I \subseteq \bigcap\{P \mid P$ is a 2-absorbing ideal and $I \subseteq P\}$. By (i), we have

$$\{P \mid P \in Spec(A) \text{ and } I \subseteq P\} \subseteq \{P \mid P \text{ is a 2-absorbing ideal and } I \subseteq P\}.$$

Then

$$\bigcap\{P \mid P \text{ is a 2-absorbing ideal and } I \subseteq P\} \subseteq \bigcap\{P \mid P \in Spec(A) \text{ and } I \subseteq P\}.$$

It follows from Theorem 2.10 that $\bigcap\{P \mid P$ is a 2-absorbing ideal and $I \subseteq P\} \subseteq I$. Therefore

$$I = \bigcap\{P \mid P \text{ is a 2-absorbing ideal and } I \subseteq P\}.$$

(*iv*) By Remark 2.2 and Theorem 2.17, it is clear.
(*v*) Suppose $a \wedge b \wedge c \in A \setminus \{1\}$ for some $a, b, c \in A$. By contrary, assume that $a \wedge b = 1$, $a \wedge c = 1$ and $b \wedge c = 1$. Since $a \wedge b \leq a \vee b$, so $a \vee b = 1$. By hypothesis $1 \neq (a \wedge b) \wedge c = 1 \wedge c = (a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c) = 1 \vee 1 = 1$, which is a contradiction. □

Corollary 3.4. (*i*) *Every maximal ideal of A is a* 2*-absorbing ideal.*
(*ii*) *If I is an obstinate ideal of A, then I is a* 2*-absorbing ideal.*
(*iii*) *Let A be an MV-chain. Then every proper ideal of A is a* 2*-absorbing ideal.*
(*iv*) *Let I be an implicative and maximal ideal. Then I is a* 2*-absorbing ideal.*

Remark 3.5. (*i*) *In general, the converse of Proposition 3.3* (*i*)*, is not true. In Example 3.2* (*i*)*, I_0 is a* 2*-absorbing ideal, but is not a prime ideal.*
(*ii*) *In Example 3.2* (*i*)*, I_0 is a* 2*-absorbing ideal but is not a maximal ideal.*
(*iii*) *It is easy to verify that I_1 in Example 3.2* (*i*)*, is a* 2*-absorbing ideal; however, it is not an obstinate ideal.*
(*iv*) *In Example 3.2* (*i*)*, I_0 is a* 2*-absorbing ideal, but is not a primary ideal. Because $a \odot d = 0 \in I_0$, but $a^n, d^n \notin I_0$, for each $n \in \mathbb{N}$.*
(*v*) *If every proper ideal of A is a* 2*-absorbing ideal, then A is not necessarily a chain. This is illustrated by Example 3.2* (*i*)*.*
(*vi*) *In general, the converse of Proposition 3.3* (*iv*)*, is not true. In Example 3.2, I_0 is a* 2*-absorbing ideal, but A/I_0 is not a chain. Hence A/I_0 is not locally finite.*
(*vii*) *The zero ideal is generally not a* 2*-absorbing ideal, as shown in Example 3.2* (*iii*)*. However, by Corollary 3.4, we can conclude that if A is an MV-chain, then the zero ideal is a* 2*-absorbing ideal.*
(*viii*) *In Example 3.2* (*i*)*, I_0 is a* 2*-absorbing ideal, but is not an implicative ideal. Because $1 \odot (b^* \odot c^*) = 0 \in I_0$ and $b \odot c^* = 0 \in I_0$, but $1 \odot c^* \notin I_0$.*

We recall that intersection two prime ideals is not necessarily a prime ideal. In the following proposition, we show that intersection two prime ideals is always a 2-absorbing ideal.

Proposition 3.6. *If P and Q are prime ideals of A, then $P \cap Q$ is a* 2*-absorbing ideal.*

Proof. Assume that $a, b, c \in A$ whenever $a \wedge b \wedge c \in P \cap Q$ and $a \wedge c, b \wedge c \notin P \cap Q$. We have $(a \wedge b) \wedge c \in P \cap Q$ hence $(a \wedge b) \wedge c \in P$ and $(a \wedge b) \wedge c \in Q$. Since P and Q are prime ideal, so $a \wedge b \in P$ or $c \in P$ and $a \wedge b \in Q$ or $c \in Q$. We have four cases:
Case 1: If $a \wedge b \in P$ and $a \wedge b \in Q$, then $a \wedge b \in P \cap Q$.
Case 2: Let $a \wedge b \in P$ and $c \in Q$. Then $a \in P$ or $b \in P$. Suppose that $a \in P$, hence $a \wedge c \in P$. Since $c \in Q$, so $a \wedge c \in Q$. Therefore $a \wedge c \in P \cap Q$, which is a contradiction. If $b \in P$ and $c \in Q$, then $b \wedge c \in P$ and $b \wedge c \in Q$. Hence $b \wedge c \in P \cap Q$, which is a contradiction.
Case 3: This case is similar to case 2.
Case 4: If $c \in P$ and $c \in Q$, then $c \in P \cap Q$. Furthermore, $a \wedge c, b \wedge c \in P \cap Q$, which is a contradiction. □

Remark 3.7. (*i*) *The converse of the previous proposition is not generally true. For instance, in Example 3.2* (*i*), $I_0 \cap I_1$ *is a* 2*-absorbing ideal, while* I_0 *is not prime.*
(*ii*) *Both* I_1 *and* I_2 *in Example 3.2* (*i*), *are prime ideals, and* $I_1 \cap I_2 = \{0\}$. *However, the intersection is* 2*-absorbing but not prime.*
(*iii*) *The intersection of three prime ideals is not necessarily* 2*-absorbing. This can be demonstrated in the example below.*

Example 3.8. *Let* $X = \mathbb{R}$. *By Theorem 2.19, we have* M_1, M_2 *and* M_3 *are maximal ideals of* $C(X)$. *It follows from Propositions 3.3 and 3.6 that* $M_1 \cap M_2$ *and* M_3 *are* 2*-absorbing ideals of* $C(X)$. *We want to show that* $(M_1 \cap M_2) \cap M_3$ *is not a* 2*-absorbing ideal. So it can be conclude that the intersection of* 2*-absorbing ideals is not necessarily a* 2*-absorbing ideal.*
Obviously, $(M_1 \cap M_2) \cap M_3 = \{k \in C(X) : \{1, 2, 3\} \subseteq Z(k)\}$. *Put*

$$f(x) = \begin{cases} 1 & x \in (-\infty, 0] \cup [2, \infty) \\ -x + 1 & x \in (0, 1] \\ x - 1 & x \in (1, 2) \end{cases}$$

$$g(x) = \begin{cases} 1 & x \in (-\infty, 1] \cup [3, \infty) \\ -x + 2 & x \in (1, 2] \\ x - 2 & x \in (2, 3) \end{cases}$$

$$h(x) = \begin{cases} 1 & x \in (-\infty, 2] \cup [4, \infty) \\ -x + 3 & x \in (2, 3) \\ x - 3 & x \in [3, 4) \end{cases}$$

Obviously, $Z(f \wedge g \wedge h) = \{1, 2, 3\}$ *then* $f \wedge g \wedge h \in (M_1 \cap M_2) \cap M_3$. *But* $Z(f \wedge g) = \{1, 2\}, Z(g \wedge h) = \{2, 3\}$ *and* $Z(f \wedge h) = \{1, 3\}$, *so* $f \wedge g, g \wedge h$ *and* $f \wedge h$ *do not belong to* $(M_1 \cap M_2) \cap M_3$. *Therefore* $(M_1 \cap M_2) \cap M_3$ *is not a* 2*-absorbing ideal.*

Remark 3.9. *Intersection (meet) and join of any family of* 2*-absorbing ideals are not necessarily* 2*-absorbing ideals. We can see in Example 3.8. Also, in Example 3.2* (*i*), $I_1 \vee I_2 = A$ *is not a* 2*-absorbing ideal.*

Theorem 3.10. *If all* 2*-absorbing ideals of* A *are maximal, then* A *has at most one prime ideal and this ideal is a maximal ideal.*

Proof. Assume that P_1 and P_2 are prime ideals of A. It follows from Proposition 3.6, that $P_1 \cap P_2$ is a 2-absorbing ideal. Also, $P_1 \cap P_2$ is a maximal ideal and is contained in both, therefore $P_1 = P_2$. □

Proposition 3.11. *Let* I *be a proper ideal of* A *and let* P *be a prime ideal of* A *such that* $I \subseteq P$. *Then* $P \in Min(I)$ *if and only if for each* $a \in P$ *there exists* $b \in A \setminus P$ *such that* $a \wedge b \in I$.

Proof. Assume that for each $a \in P$, there exists $b \in A \setminus P$ such that $a \wedge b \in I$. We show that $P \in Min(I)$. By contrary, suppose there exists $Q \in Min(I)$ such that $Q \subsetneq P$. Thus, there exist $a \in P \setminus Q$ and $b \in A \setminus P$ such that $a \wedge b \in I \subseteq Q$. Hence $a \wedge b \in Q$, since Q is prime and $a \notin Q$, it follows that $b \in Q$. Consequently, $b \in P$, which leads to a contradiction.
Conversely, assume that there exists $a \in P$ such that for each $b \in A \setminus P, a \wedge b \notin I$. Define the set $S = \{a \wedge b \mid b \in A \setminus P\} \cup \{1\}$. This set S is a $\wedge$-closed system in A. So there exists $K \in Spec(A)$ such that $K \cap S = \emptyset$. If $K \subseteq P$, since P is minimal over I, so $K = P$ and $a \in K$. However, since $a \wedge 1 = a \in S$, hence $a \in K \cap S$, which is a contradiction. Now, if $K \nsubseteq P$, then there exists $x \in K \setminus P$. We know that $a \wedge x \leq x$, hence $a \wedge x \in K$. Clearly, $a \wedge x \in S$, therefore $K \cap S \neq \emptyset$, which again results in a contradiction. □

Theorem 3.12. *Let* I *be a* 2*-absorbing ideal of* A. *Then* $|Min(I)| \leq 2$.

Proof. We denote $T = \{P_i \mid P_i \in Min(I)\}$. By Theorems 2.9 and 2.14, it follows that T is nonempty. We proceed by contradiction, suppose T has at least three elements. Assume that $P_1, P_2 \in T$ are two distinct prime ideals. So there exists $a_1 \in P_1 \setminus P_2$ and there exists $a_2 \in P_2 \setminus P_1$. We show that $a_1 \wedge a_2 \in I$. It follows from Theorem 3.11, that there exists $r_2 \notin P_1$ and there is an $r_1 \notin P_2$ such that $a_1 \wedge r_2 \in I$ and $a_2 \wedge r_1 \in I$. Obviously, $a_1, a_2 \notin P_1 \cap P_2$. Since $a_1 \wedge r_2, a_2 \wedge r_1 \in I \subseteq P_1 \cap P_2$ and P_1, P_2 are prime ideals, we conclude that $r_1 \in P_1 \setminus P_2$ and $r_2 \in P_2 \setminus P_1$. Thus $r_1, r_2 \notin P_1 \cap P_2$ and we obtain that $r_1 \vee r_2 \notin P_1 \cap P_2$. (Because $r_1, r_2 \le r_1 \vee r_2$ and if $r_1 \vee r_2 \in P_1 \cap P_2$, then $r_1, r_2 \in P_1 \cap P_2$, which is a contradiction). Observe that $r_1 \vee r_2 \notin P_1$ and $r_1 \vee r_2 \notin P_2$. Since $a_1 \wedge r_2 \in I$ and $a_2 \wedge r_1 \in I$, so by Remark 2.6, we get $(a_2 \wedge r_1) \vee (a_1 \wedge r_2) \in I$. By Theorem 2.3, $(a_1 \wedge a_2) \wedge (r_1 \vee r_2) = (a_1 \wedge a_2 \wedge r_1) \vee (a_1 \wedge a_2 \wedge r_2) \le (a_2 \wedge r_1) \vee (a_1 \wedge r_2)$ and we conclude that $(a_1 \wedge a_2) \wedge (r_1 \vee r_2) \in I$. Since $(r_1 \vee r_2) \wedge a_1 \notin P_2$ and $(r_1 \vee r_2) \wedge a_2 \notin P_1$, it follows that neither $(r_1 \vee r_2) \wedge a_1 \in I$ nor $(r_1 \vee r_2) \wedge a_2 \in I$. Given that I is a 2-absorbing ideal, we conclude that $a_1 \wedge a_2 \in I$. Now suppose there exists $P_3 \in T$ such that P_3 is neither P_1 nor P_2. We can choose $x_1 \in P_1 \setminus (P_2 \cup P_3)$, $x_2 \in P_2 \setminus (P_1 \cup P_3)$ and $x_3 \in P_3 \setminus (P_1 \cup P_2)$. By the previous argument, we get $x_1 \wedge x_2 \in I$. Also, since $I \subseteq P_1 \cap P_2 \cap P_3$, so $x_1 \wedge x_2 \in P_1 \cap P_2 \cap P_3$. We deduce that either $x_1 \in P_3$ or $x_2 \in P_3$, leading to a contradiction. Therefore T can contain at most two elements. □

Proposition 3.13. *If I is a 2-absorbing ideal of A such that I is not a prime ideal, then $|Min(I)| = 2$.*

Proof. As I is 2-absorbing, hence by Theorem 3.12, $|Min(I)| \le 2$. Assume for contradiction, $|Min(I)| \neq 2$, that is, $|Min(I)| = 1$. On the other hand $Min(I) = P$, it follows from Proposition 3.11, that for each $a \in P$ there exists $b \in A \setminus P$ such that $a \wedge b \in I$. Since I is not prime, so $a, b \notin I$. We have $a \notin I$, hence there exists $Q \in Spec(A)$ such that $I \subseteq Q$ and $a \notin Q$. By Theorem 2.14, we deduce that there exists $P^* \in Min(I)$ such that $P^* \subseteq Q$. Since obviously, $P \neq P^*$, which contradicts the hypothesis that $|Min(I)| = 1$. □

Remark 3.14. *In Example 3.2 (i), I_0 is a 2-absorbing ideal, but is not prime ideal, and $|Min(I_0)| = 2$. In the same example, for the ideal I_2, we have $|Min(I_2)| = 1$.*

Lemma 3.15. *Let I be a 2-absorbing ideal of A. Suppose that $(a] \cap (b] \cap J \subseteq I$ for some $a, b \in A$ and an ideal J of A. If $a \wedge b \notin I$, then either $(a] \cap J \subseteq I$ or $(b] \cap J \subseteq I$.*

Proof. Assume that $(a] \cap J \nsubseteq I$ and $(b] \cap J \nsubseteq I$. Hence, there are some $x, y \in J$ such that $a \wedge x \notin I$ and $b \wedge y \notin I$. Since $a \wedge b \wedge x \in I$ and $a \wedge b \notin I$ and $a \wedge x \notin I$, so $b \wedge x \in I$. Also, $a \wedge b \wedge y \in I$ and $a \wedge b \notin I$ and $b \wedge y \notin I$, thus $a \wedge y \in I$. We have $a \wedge b \wedge y \in I$ and $a \wedge b \wedge x \in I$, so $(a \wedge b \wedge y) \oplus (a \wedge b \wedge x) \in I$. Since I is an ideal and by Theorem 2.3, we conclude that $(a \wedge b) \wedge (x \oplus y) \in I$. By hypothesis $a \wedge b \notin I$ and I is a 2-absorbing ideal, so $a \wedge (x \oplus y) \in I$ or $b \wedge (x \oplus y) \in I$. If $a \wedge (x \oplus y) \in I$, then $a \wedge x \in I$ (because $a \wedge x \le a \wedge (x \oplus y)$), which is a contradiction. Similarly, if $b \wedge (x \oplus y) \in I$, then $b \wedge y \in I$ (because $b \wedge y \le b \wedge (x \oplus y)$), which is again a contradiction. □

Next, we present an equivalent definition of a 2-absorbing ideal.

Theorem 3.16. *Let I be a proper ideal of A. Then I is a 2-absorbing ideal of A if and only if whenever $I_1 \cap I_2 \cap I_3 \subseteq I$ for some ideals I_1, I_2, I_3 of A, then either $I_1 \cap I_2 \subseteq I$ or $I_2 \cap I_3 \subseteq I$ or $I_1 \cap I_3 \subseteq I$.*

Proof. First we show that I is a 2-absorbing ideal. Suppose $I_1 \cap I_2 \cap I_3 \subseteq I$ for some ideals I_1, I_2, I_3 of A, then $I_1 \cap I_2 \subseteq I$ or $I_2 \cap I_3 \subseteq I$ or $I_1 \cap I_3 \subseteq I$. Let $a, b, c \in A$ and $a \wedge b \wedge c \in I$. Assume also that $a \wedge b \notin I$ and $b \wedge c \notin I$. Let $I_1 = (a]$, $I_2 = (b]$ and $I_3 = (c]$. Then $I_1 \cap I_2 \cap I_3 = (a] \cap (b] \cap (c] = (a \wedge b \wedge c] \subseteq I$. Since $I_1 \cap I_2 \nsubseteq I$ and $I_2 \cap I_3 \nsubseteq I$, so $I_1 \cap I_3 = (a \wedge c] \subseteq I$; that is, $a \wedge c \in I$.
Conversely, assume that I is a 2-absorbing ideal of A and $I_1 \cap I_2 \cap I_3 \subseteq I$ for some ideals I_1, I_2, I_3 of A, such that $I_1 \cap I_2 \nsubseteq I$. We show that $I_2 \cap I_3 \subseteq I$ or $I_1 \cap I_3 \subseteq I$. By contrary, if $I_2 \cap I_3 \nsubseteq I$ and $I_1 \cap I_3 \nsubseteq I$. So there exist $a_1 \in I_1$ and $a_2 \in I_2$ such that $(a_1] \cap I_3 \nsubseteq I$ and $(a_2] \cap I_3 \nsubseteq I$. Also, $(a_1] \cap (a_2] \cap I_3 \subseteq I$ and $(a_1] \cap I_3 \nsubseteq I$ and $(a_2] \cap I_3 \nsubseteq I$, it follows from Lemma 3.15, that $a_1 \wedge a_2 \in I$. Since $I_1 \cap I_2 \nsubseteq I$, so there are $a \in I_1$, $b \in I_2$, such that $a \wedge b \notin I$. As $(a] \cap (b] \cap I_3 \subseteq I$ and $a \wedge b \notin I$, by Lemma 3.15, we have $(a] \cap I_3 \subseteq I$ or $(b] \cap I_3 \subseteq I$. Here three cases arise.
Case 1: Suppose $(a] \cap I_3 \subseteq I$, but $(b] \cap I_3 \nsubseteq I$. As $(a_1] \cap (b] \cap I_3 \subseteq I$ and $(b] \cap I_3 \nsubseteq I$ and $(a_1] \cap I_3 \nsubseteq I$, hence $(a_1] \cap (b] \subseteq I$. On the other hand, $a_1 \wedge b \in I$. Also, since $a_1, a \in I_1$, so $a_1 \oplus a \in I_1$ and we deduce that $(a_1] \vee (a] = (a_1 \oplus a] \in I_1$. Since $(a] \cap I_3 \subseteq I$ and $(a_1] \cap I_3 \nsubseteq I$, so $((a_1] \vee (a]) \cap I_3 \nsubseteq I$. Now, we have

$((a_1] \vee (a]) \cap (b] \cap I_3 \subseteq I$ and $(b] \cap I_3 \nsubseteq I$ and $((a_1] \vee (a]) \cap I_3 \nsubseteq I$, again by Lemma 3.15, we conclude that $((a_1] \vee (a]) \cap (b] \subseteq I$. On the other hand, $(a_1 \oplus a) \wedge b \in I$, we know $a \wedge b \leq (a_1 \oplus a) \wedge b$, therefore $a \wedge b \in I$, which is a contradiction.

Case 2: Suppose $(b] \cap I_3 \subseteq I$, but $(a] \cap I_3 \nsubseteq I$. Since $a_2, b \in I_2$, so $a_2 \oplus b \in I_2$ and by Theorem 2.7, $(a_2] \vee (b] \in I_2$. Also, $(b] \cap I_3 \subseteq I$ and $(a_2] \cap I_3 \nsubseteq I$, we deduce that $((b] \vee (a_2]) \cap I_3 \nsubseteq I$. We have $(a] \cap ((b] \vee (a_2]) \cap I_3 \subseteq I$ and $((b] \vee (a_2]) \cap I_3 \nsubseteq I$ and $(a] \cap I_3 \nsubseteq I$, it follows from Lemma 3.15, that $(a] \cap ((a_2] \vee (b]) \in I$. We get $a \wedge (a_2 \oplus b) \in I$ and so $a \wedge b \in I$, which is a contradiction.

Case 3: Suppose $(b] \cap I_3 \subseteq I$ and $(a] \cap I_3 \subseteq I$. We know that $(a_2] \cap I_3 \nsubseteq I$ and $(b] \cap I_3 \subseteq I$, hence $((b] \vee (a_2]) \cap I_3 \nsubseteq I$. Also, $(a] \cap I_3 \subseteq I$ and $(a_1] \cap I_3 \nsubseteq I$, thus we deduce that $((a_1] \vee (a]) \cap I_3 \nsubseteq I$. Since $((a] \vee (a_1]) \cap ((b] \vee (a_2]) \cap I_3 \subseteq I$ and $((a] \vee (a_1]) \cap I_3 \nsubseteq I$ and $((b] \vee (a_2]) \cap I_3 \nsubseteq I$ by Lemma 3.15 we conclude that, $((a] \vee (a_1]) \cap ((b] \vee (a_2]) \subseteq I$. Thus we get $(a \oplus a_1) \wedge (b \oplus a_2) \in I$. It is clear that $a \wedge b \leq (a \oplus a_1) \wedge b \leq (a \oplus a_1) \wedge (b \oplus a_2)$. Therefore $a \wedge b \in I$, which is a contradiction. □

Example 3.17. *Let ${}^*\mathbb{R}$ be a non-standard model of real numbers with natural order and ε be a positive infinitesimal element of ${}^*\mathbb{R}$. Let $\varepsilon^2 = \varepsilon \cdot \varepsilon, ..., \varepsilon^n = \varepsilon \cdot \varepsilon \cdot ... \cdot \varepsilon(n - times)$, where $\cdot$ is the usual product in the field ${}^*\mathbb{R}$; then $\varepsilon^i > 0$ for any $i \in \mathbb{N}$ and $\varepsilon^i \ll \varepsilon^j$, for $i > j$.*
The unit interval ${}^*[0, 1] \subseteq^* \mathbb{R}$ is an semilocal MV-algebra with the operations: $x \oplus y = min\{1, x + y\}, x^* = 1 - x$. Let $\mathbb{N}$ be the ordered set of positive natural numbers. For every $n \in \mathbb{N}$, let E_n be the subalgebra of ${}^*[0, 1]$ generated by $\{\varepsilon, \varepsilon^2, ..., \varepsilon^n\}$ and E be the subalgebra $\bigcup_{n\in\mathbb{N}} E_n$ generated by $\{\varepsilon, \varepsilon^2, ..., \varepsilon^n, \ldots\}$ ([8]). The ideals of E are $\{0\}, (\varepsilon], ..., (\varepsilon^i]...$, where $i \in \mathbb{N}$ and $(\varepsilon^i] \subseteq (\varepsilon^j]$, for any $i > j$. $I = (\varepsilon^3]$ is a 2-absorbing ideal. Consider: $I_1 = (\varepsilon^2]$, $I_2 = (\varepsilon]$ and $I_3 = (\varepsilon^5]$. It is clear that $I_1 \cap I_2 \cap I_3 \subseteq I$ and $I_2 \cap I_3 \subseteq I$ and $I_1 \cap I_3 \subseteq I$.

Remark 3.18. *Let I_1, I_2 and I be ideals of A and let $I \subseteq I_1 \cup I_2$. Then $I \subseteq I_1$ or $I \subseteq I_2$.*

Proposition 3.19. *If I is a 2-absorbing ideal of A, then for all $a, b \in A$ such that $a \wedge b \notin I$, $(I : a \wedge b) \subseteq (I : a)$ or $(I : a \wedge b) \subseteq (I : b)$.*

Proof. Suppose that $a \wedge b \notin I$ where $a, b \in A$ and $t \in (I : a \wedge b)$. Hence $t \wedge a \wedge b \in I$. By hypothesis I is a 2-absorbing ideal and $a \wedge b \notin I$, thus either $t \wedge a \in I$ or $t \wedge b \in I$. We deduce that $t \in (I : a)$ or $t \in (I : b)$. Therefore, $(I : a \wedge b) \subseteq (I : a) \cup (I : b)$. By Remark 3.18, we conclude that $(I : a \wedge b) \subseteq (I : a)$ or $(I : a \wedge b) \subseteq (I : b)$. □

Proposition 3.20. *If I is a 2-absorbing ideal of A, then $(I : x)$ is a 2-absorbing ideal of A for all $x \in A \setminus I$.*

Proof. Let a, b, c be elements of A such that $a \wedge b \wedge c \in (I : x)$. Then $a \wedge b \wedge c \wedge x = a \wedge (b \wedge c) \wedge x \in I$. Since I is a 2-absorbing ideal of A, so $a \wedge x \in I$ or $b \wedge c \wedge x \in I$ or $a \wedge b \wedge c \in I$. If $b \wedge c \wedge x \in I$, we are done. If $a \wedge x \in I$, since $a \wedge b \wedge x \leq a \wedge x$, then $x \wedge a \wedge b \in I$. If $a \wedge b \wedge c \in I$, then $a \wedge b \in I$ or $a \wedge c \in I$ or $b \wedge c \in I$, which implies $x \wedge a \wedge b \in I$ or $x \wedge a \wedge c \in I$ or $x \wedge b \wedge c \in I$. □

The converse of the previous proposition is true when $(I : x)$ is a prime ideal, for all $x \in A \setminus I$.

Proposition 3.21. *Let $(I : x)$ be a prime ideal of A, for all $x \in A \setminus I$. Then I is a 2-absorbing ideal of A.*

Proof. Suppose $a \wedge b \wedge c \in I$, where $a, b, c \in A$ and $a \wedge b \notin I$, $a \wedge c \notin I$. This implies $a, b, c \notin I$. By the hypothesis, $(I : a)$, $(I : b)$ and $(I : c)$ are prime ideals. Since $a \wedge b \wedge c \in I$, so we know that $a \wedge b \in (I : c)$. Therefore, either $a \in (I : c)$ or $b \in (I : c)$. This leads to $b \wedge c \in I$. □

Remark 3.22. *By Proposition 3.20 and the fact that $(0 : x) = Ann(x)$, we conclude that if the zero ideal is a 2-absorbing ideal, then $Ann(x)$ must also be 2-absorbing, for all $x \neq 0$. However, $Ann(x)$ is not necessarily a 2-absorbing ideal. In Example 3.2 (i), it can be easily verified that $Ann(0) = A$, which is not a 2-absorbing ideals.*

Proposition 3.23. *Let I be a proper ideal of A such that if there exist proper ideals H and K whenever $H \cap K = I$, implies that $H = I$ or $K = I$. Then I is a 2-absorbing ideal of A.*

Proof. Suppose that $a \wedge b \wedge c \in I$ and $a \wedge b \notin I$. We show that $a \wedge c \in I$ or $b \wedge c \in I$. On contrary, we assume that $a \wedge c \notin I$ and $b \wedge c \notin I$. Then $K = (I \cup \{a \wedge c\}]$ and $H = (I \cup \{b \wedge c\}]$ are the ideals of A properly contain I. It is clear that $K \neq I$ and $H \neq I$, hence by hypothesis $K \cap H \neq I$. On the other hand, there exists $p \in H \cap K$ such that $p \notin I$. By Lemma 2.7, $p \leq (a_1 \oplus n(a \wedge c)) \wedge (a_2 \oplus m(b \wedge c))$, for some $a_1, a_2 \in I$ and positive integers n, m. If $b = a_1 \oplus a_2$ and $r = m + n$, then $p \leq (a_1 \oplus n(a \wedge c)) \wedge (a_2 \oplus m(b \wedge c)) \leq (b \oplus r(a \wedge c)) \wedge (b \oplus r(b \wedge c)) = b \oplus r((a \wedge c) \wedge (b \wedge c))$. Since $b \oplus r(a \wedge b \wedge c) \in I$, so we conclude that $p \in I$, which is a contradiction. □

The converse of the above proposition is generally not true. In Example 3.2 (*i*), we observe that I_0 is a 2-absorbing ideal and $I_1 \cap I_2 = I_0$, but $I_0 \neq I_1$ and $I_0 \neq I_2$.

Proposition 3.24. *Let $f : A \longrightarrow B$ be an MV-homomorphism and let J be a 2-absorbing ideal of B. Then $f^{-1}(J)$ is a 2-absorbing ideal of A.*

Proposition 3.25. *If I is a 2-absorbing ideal of A and S is a $\wedge$-closed system of A such that $I \cap S = \emptyset$, then I/S is also a 2-absorbing ideal of A/S.*

Proof. Suppose that $(a/S) \wedge (b/S) \wedge (c/S) \in I/S$, so $(a \wedge b \wedge c)/S \in I/S$. By Theorem 2.16, we get $a \wedge b \wedge c \in I$. If $(a/S) \wedge (b/S) \notin I/S$ and $(b/S) \wedge (c/S) \notin I/S$, then $(a \wedge b)/S, (b \wedge c)/S \notin I/S$. Since I is a 2-absorbing ideal, so we deduce that $a \wedge c \in I$. Therefore $(a \wedge c)/S \in I/S$. □

The relationships between 2-absorbing ideals and other ideals is shown in the diagram below.

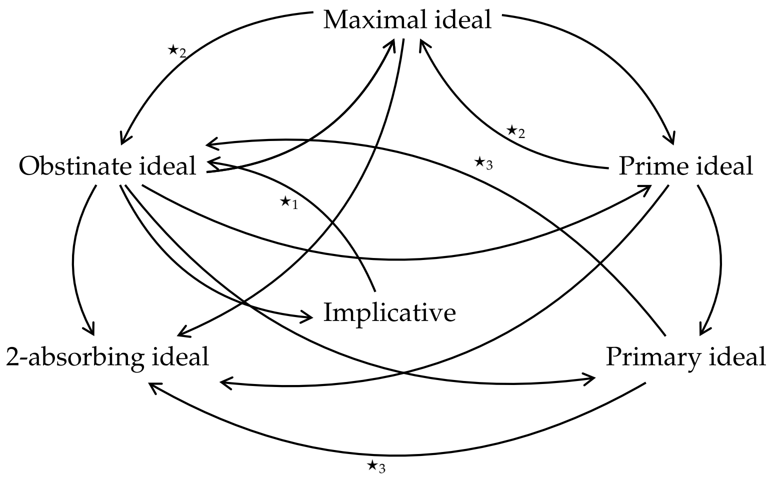

Figure 2

$\star_1$:= *Max*, $\star_2$:=Boolean ideal, $\star_3$:=Quasi implicative.

Conclusion and future research

The concept of a prime ideal plays a fundamental role in the study of MV-algebras, and the notion of 2-absorbing ideal is a generalization of it. We demonstrated that every prime ideal is a 2-absorbing ideal, and by providing an example, showed that every 2-absorbing ideal is not necessarily a prime ideal. Additionally, an example was presented illustrating that the zero ideal is not always 2-absorbing, although it can be in chains. We also proved that the intersection of two prime ideals is a 2-absorbing ideal. The number of minimal prime ideals over a 2-absorbing ideal was also examined, and it was concluded that if I is not a prime ideal, then $|Min(I)| = 2$. It was shown that every proper ideal I can be expressed as an intersection of 2-absorbing ideals, including I. For future research, we want to study the topology resulting from these ideals and obtain its properties.

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